



Modeling and Simulation of SEERS

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Plasma Modeling and Simulation

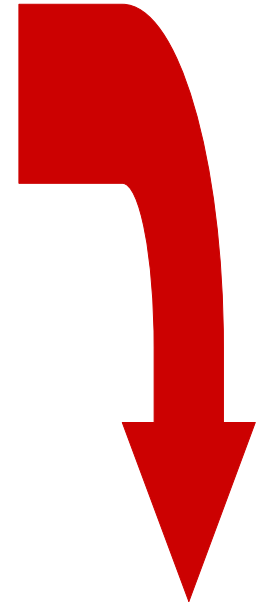
Perspectives on SEERS

Lumped Circuit Models

A General Mathematical Model

A Simplified Toy Model

Outlook



Th. Mussenbrock/R.P. Brinkmann:
The SEERS-Diagnostic Method in
in High-Density Plasmas





Plasma Modeling and Simulation

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Plasma reactors



Typical regime:

- 1 kW RF power
- 1 Pa gas pressure
- 10^{-2} m³ volume
- 10^{17} m⁻³ density

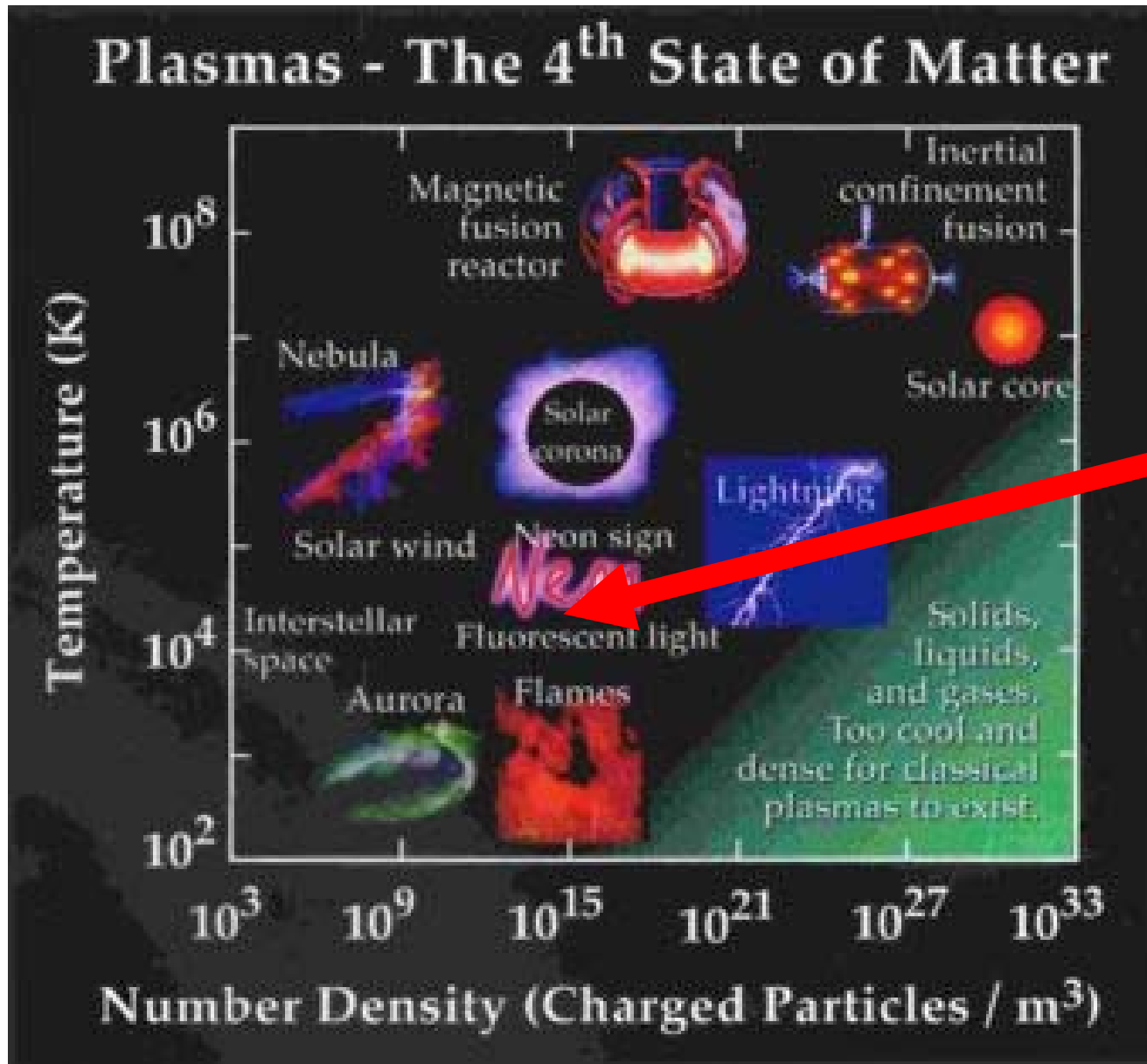
Source: Sentech



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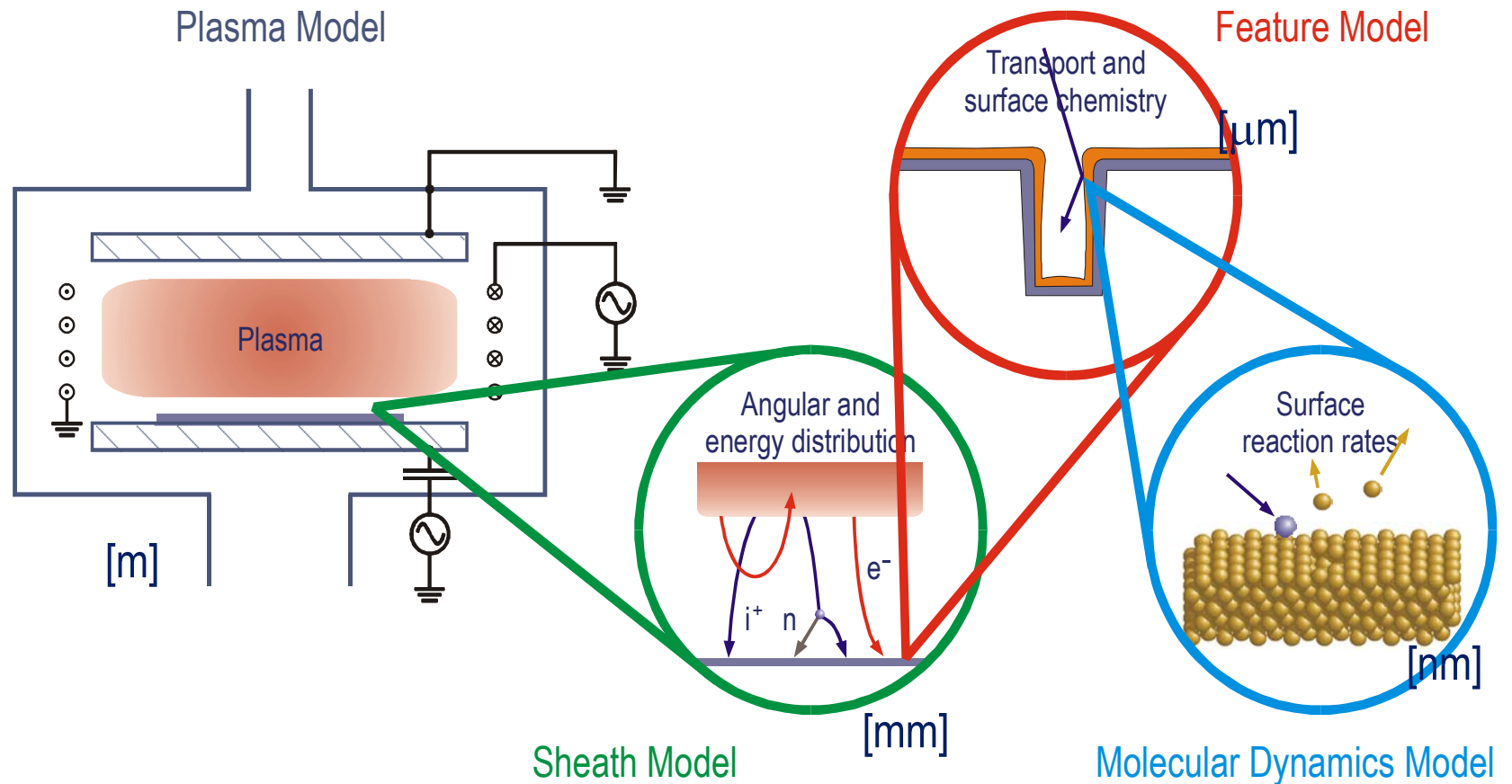
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Plasma regimes



You are here

Plasma modeling



Plasma modeling: Multiple scales, multiple physics!!!



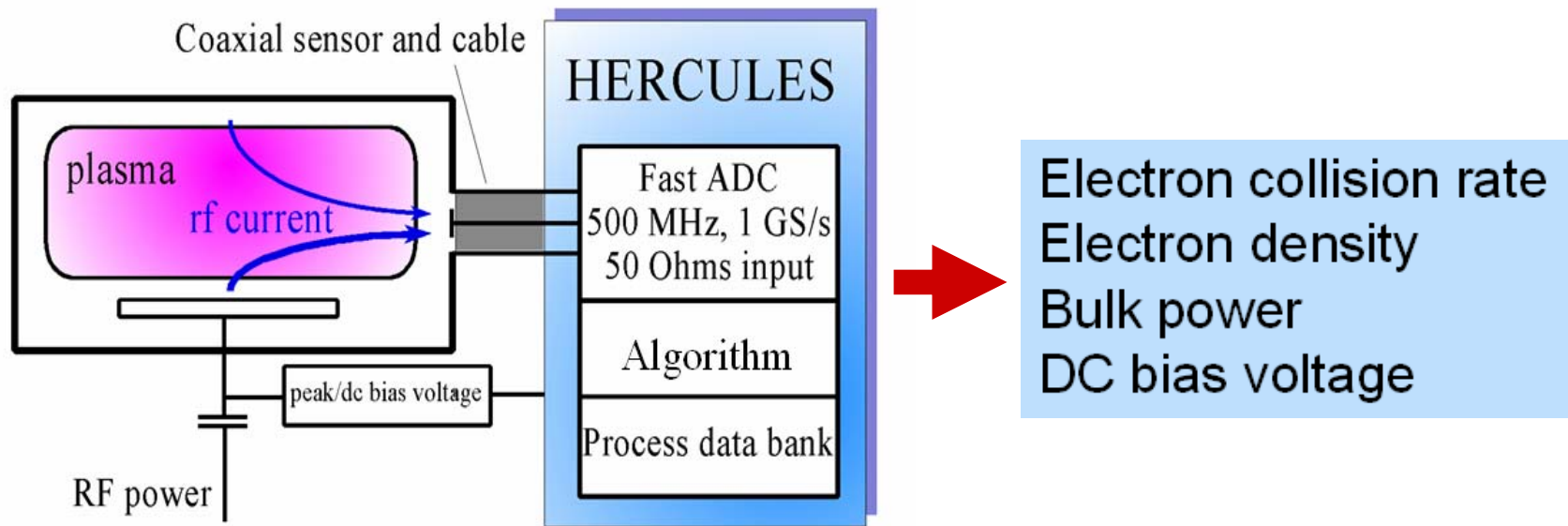


Perspectives on SEERS

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Practitioners view on SEERS

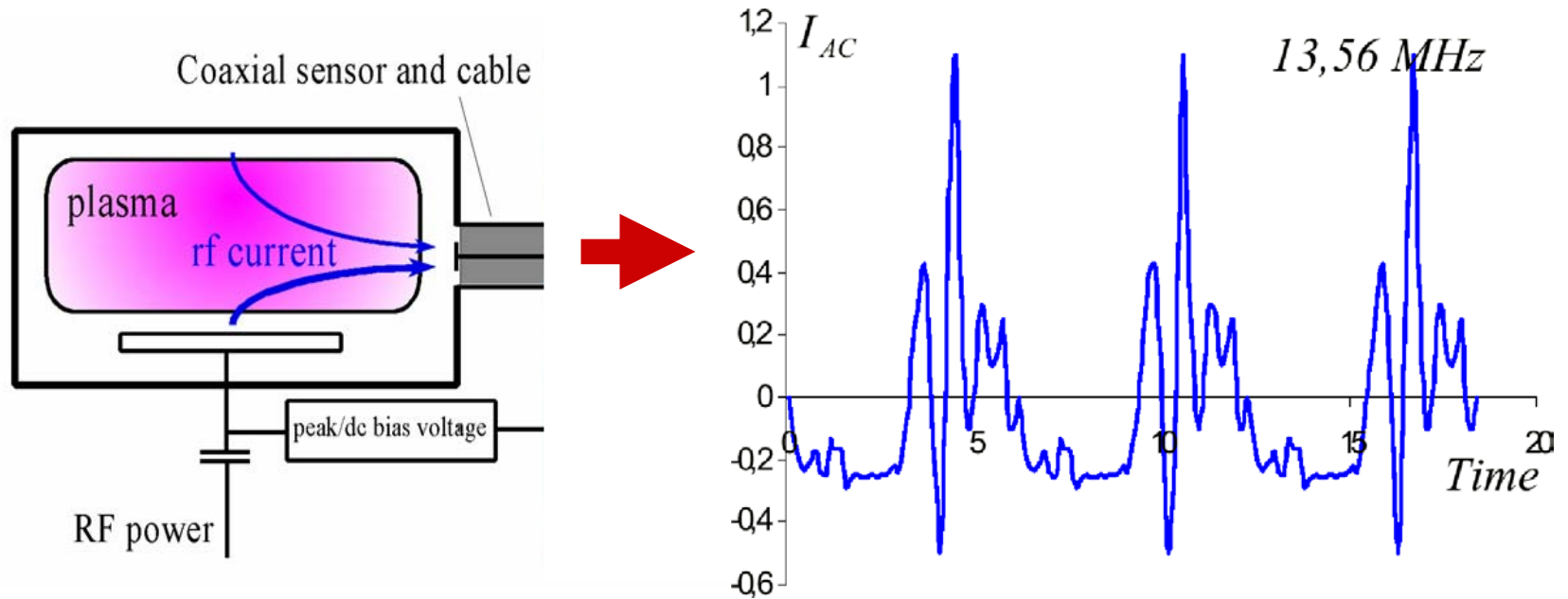


Typical points of investigation:

- How are the parameters related to the process result?
- Which of the parameters is the most sensitive?
- How does SEERS compare with other diagnostics?



Our view on SEERS

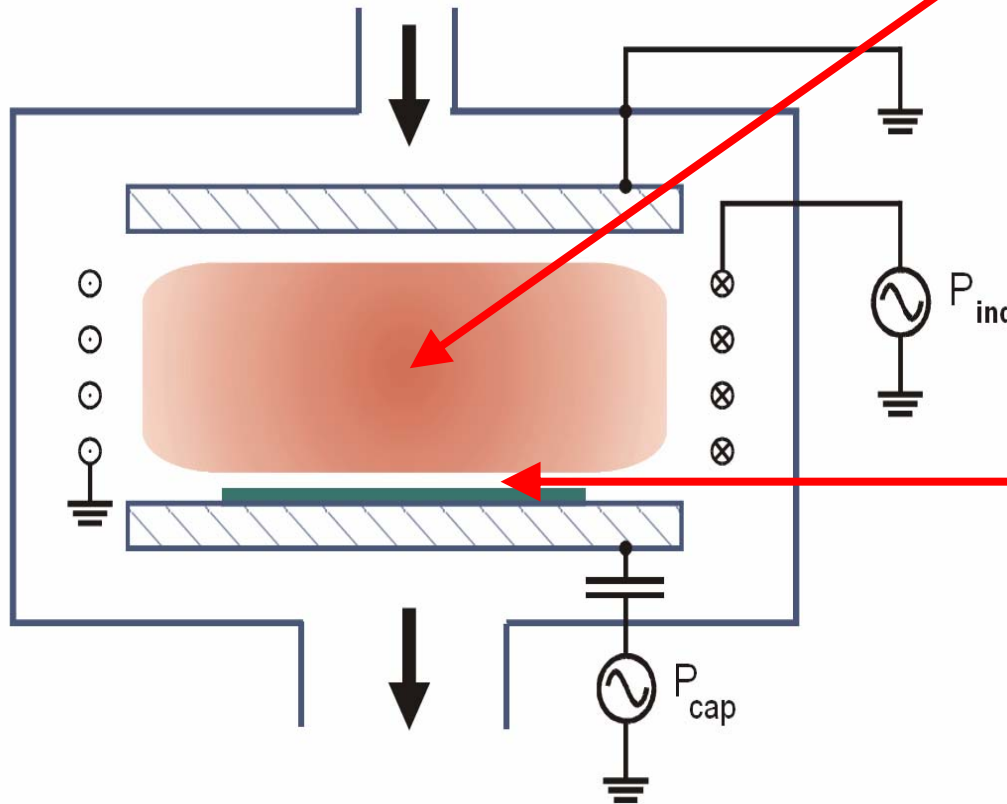


Our points of investigation:

- What is the physical background of the resonance?
- What **exactly** do the SEERS parameter mean?
- Is there more information in the SEERS signal?



Nature of the resonance



Bulk is “inductive”:

$$m \frac{\partial v}{\partial t} = -eE - \nu_c v$$

$$\leadsto \frac{\partial j}{\partial t} = -\frac{e^2 n}{m_e} E - \nu_c j$$

$$\leadsto U = L \frac{dI}{dt} + RI$$

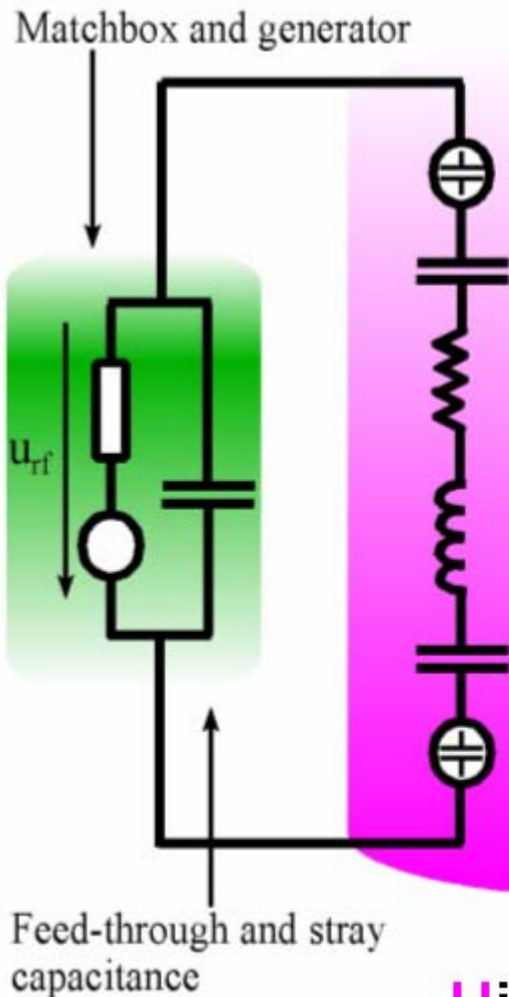
Sheath is capacitive

$$j = \epsilon_0 \frac{\partial E}{\partial t}$$

$$U = EH$$

$$\leadsto \frac{dU}{dt} = \frac{1}{C} I$$

Equivalent circuit



Hercules equivalent circuit

- $\propto u_1[-] i dt]$ nonlinearity sheath plasma-wall
- $\propto S_1$ linear 'part' of wall sheath
- $\propto \nu l m_e$ ohmic part of plasma bulk (ohmic and stochastic heating)
- $\propto \omega l m_e$ inertia (imaginary) part thereof
- $\propto S_0$ linear 'part' of rf sheath
- $\propto u_0[+] i dt]$ nonlinearity sheath rf electrode

High Frequency Electron Resonance
Current Low Pressure Spectroscopy

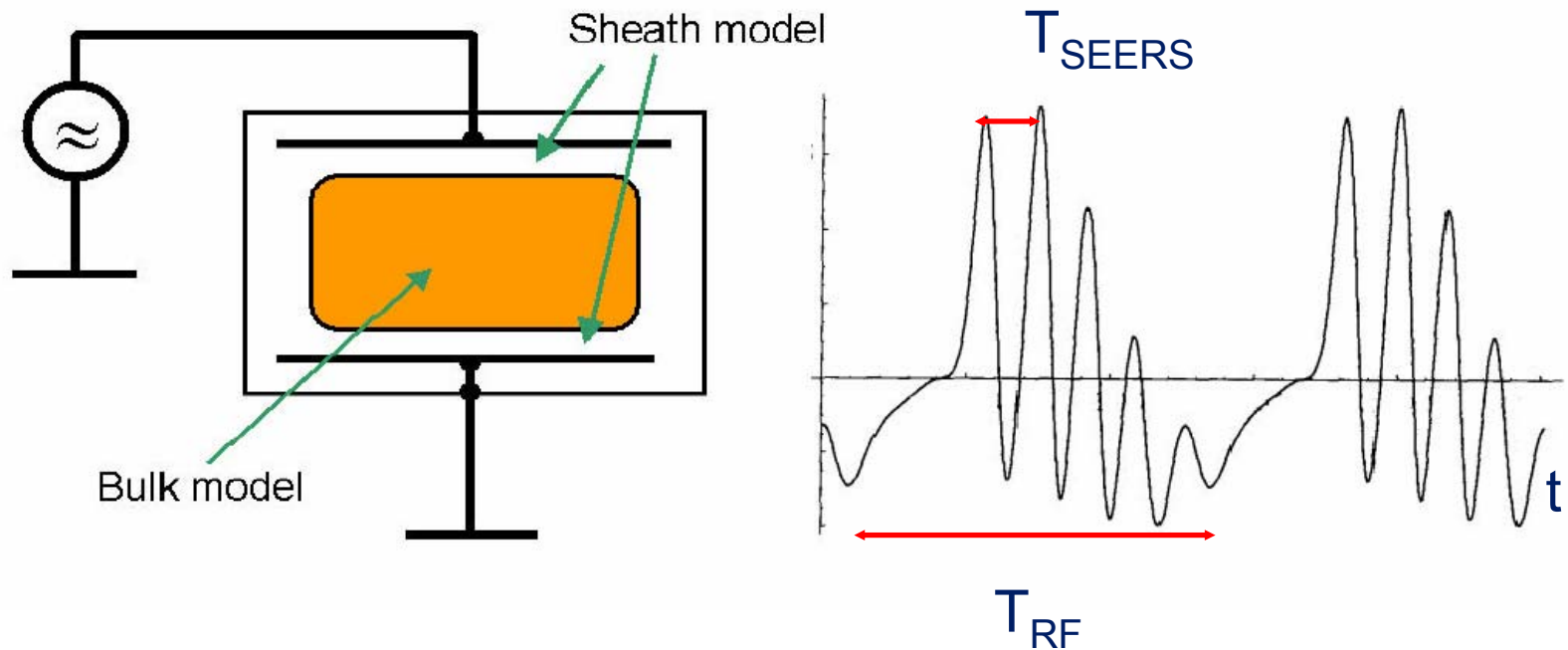


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Lumped Circuit Models

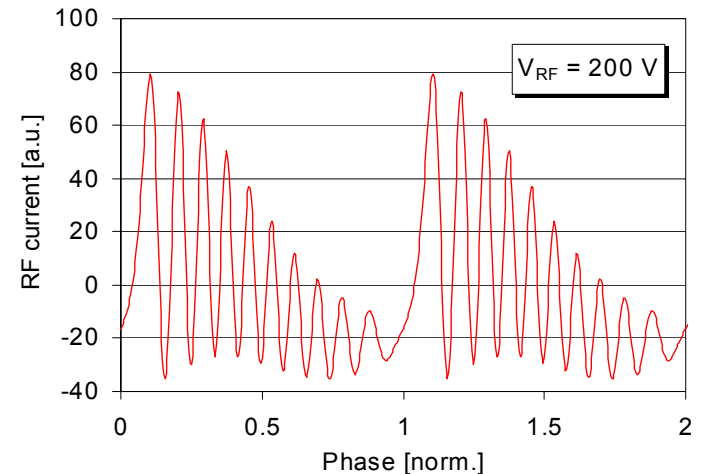
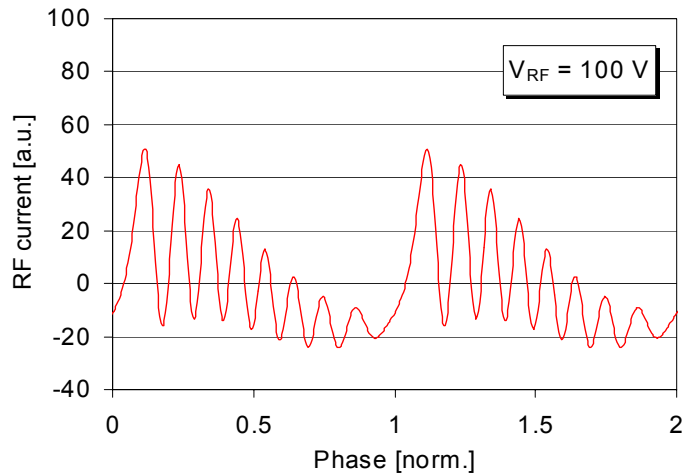
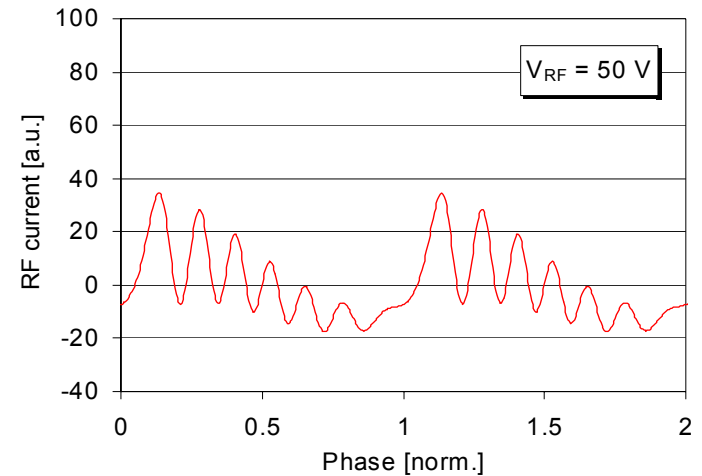
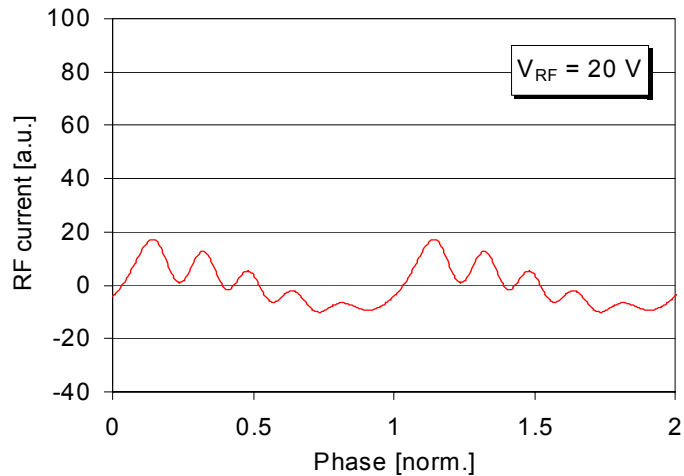
Lumped circuit model



- Linear bulk model to represent effective impedance
- One or two instances of a non-linear sheath model
- Solved either analytically or numerically



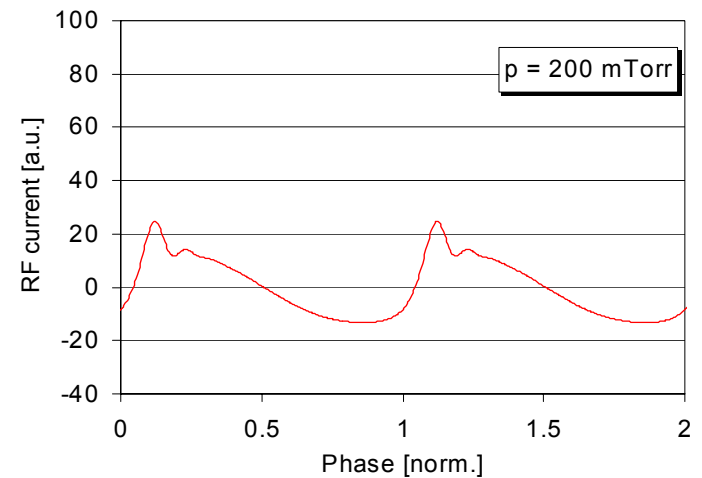
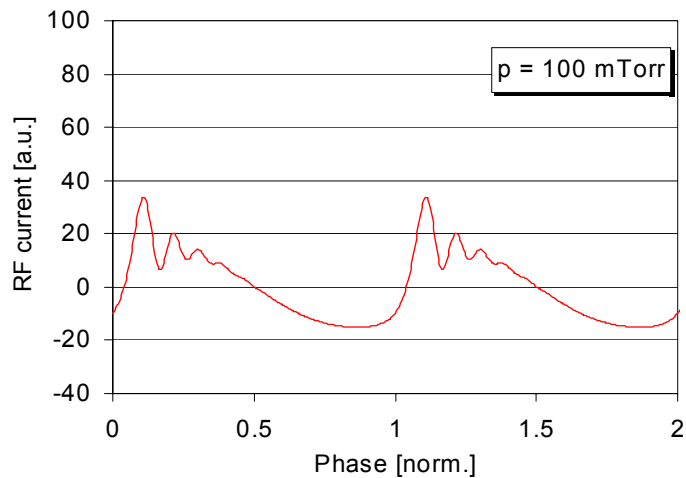
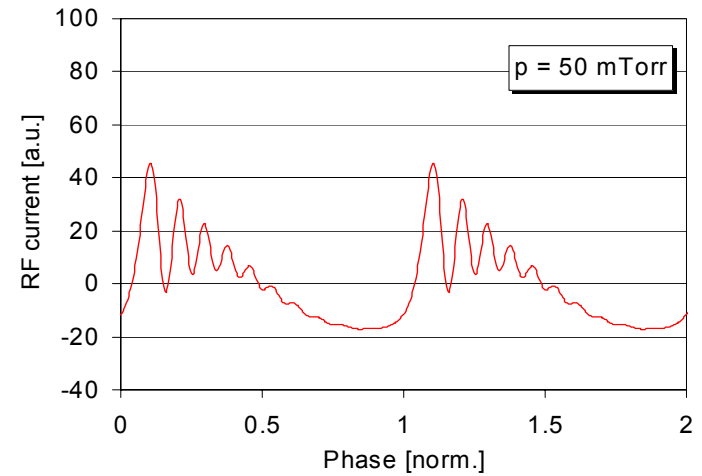
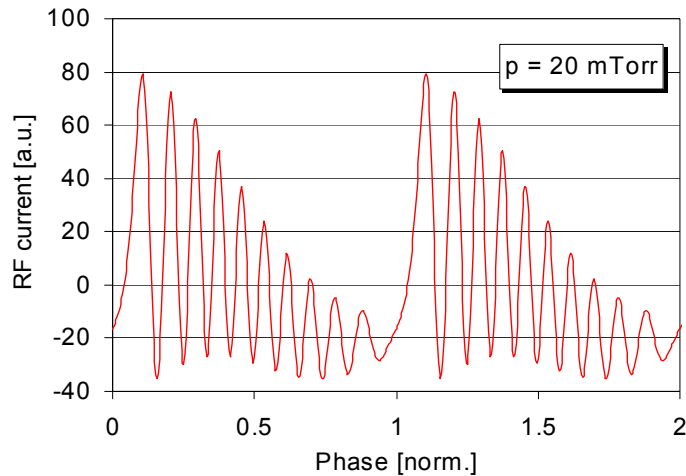
Parameter study: RF-voltage



Amplitude of the oscillation increases with excitation.



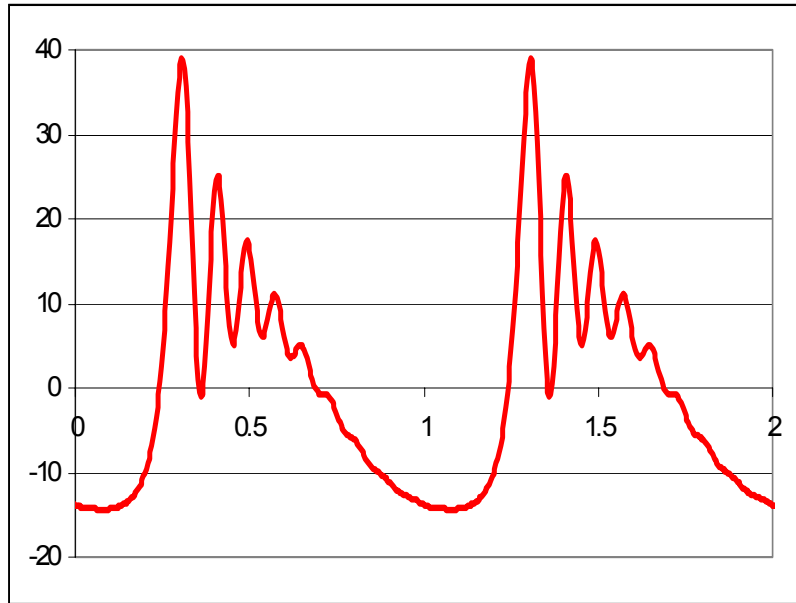
Parameter study: Pressure



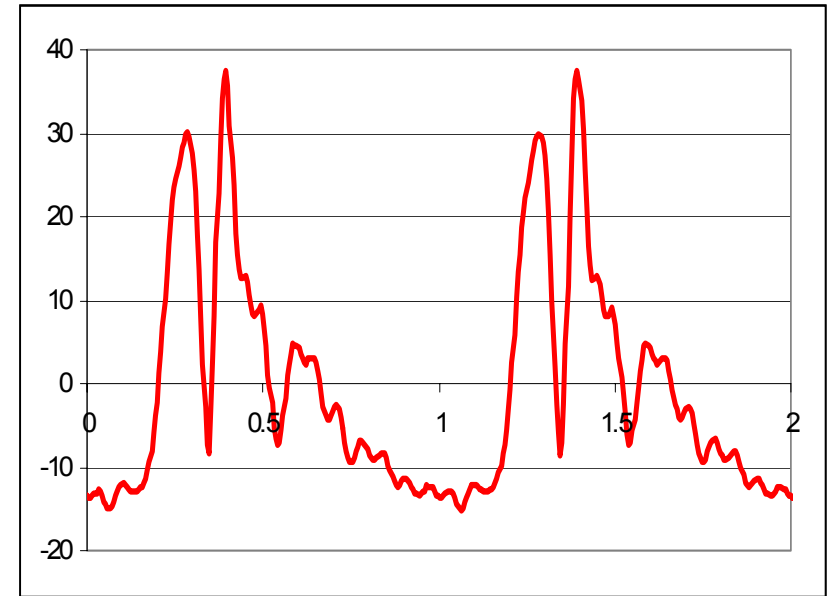
Damping increases with neutral gas pressure.



Comparing with reality



Simulation



SEERS Sensor-Data

- Good “qualitative” correspondence (form, trends).
- No quantitative agreement, no “fine structures”

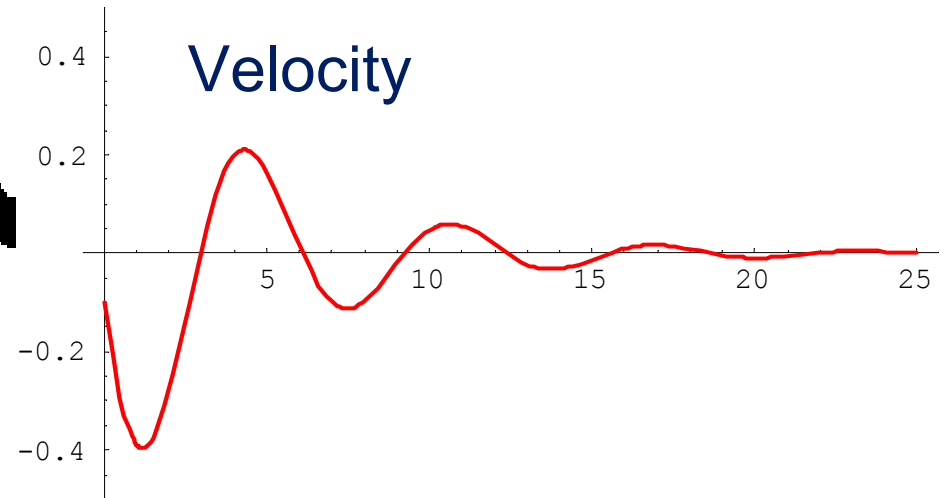
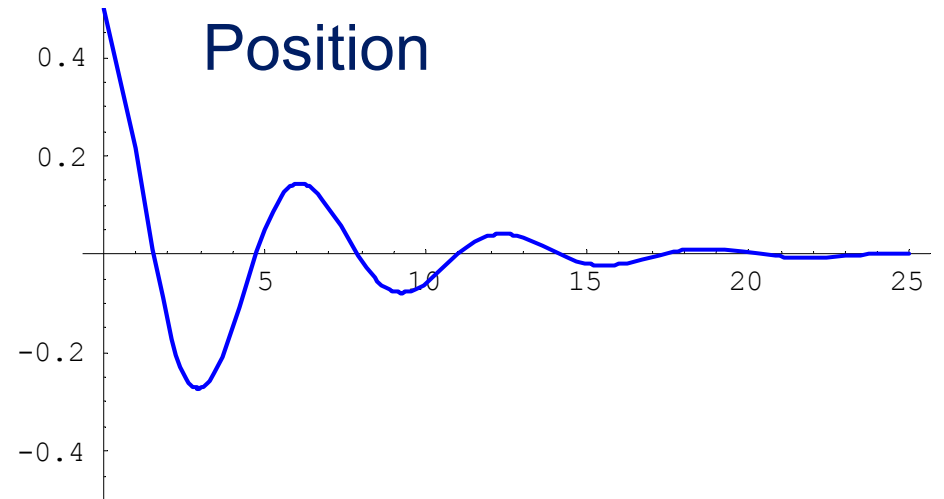
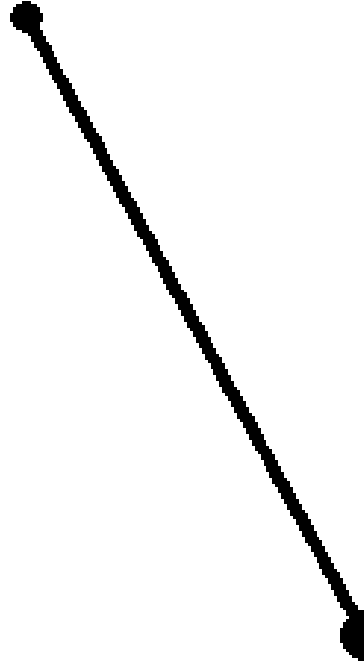
The real signal is obviously much more complex! **WHY?**





A General Mathematical Model

Analogy: Pendulum



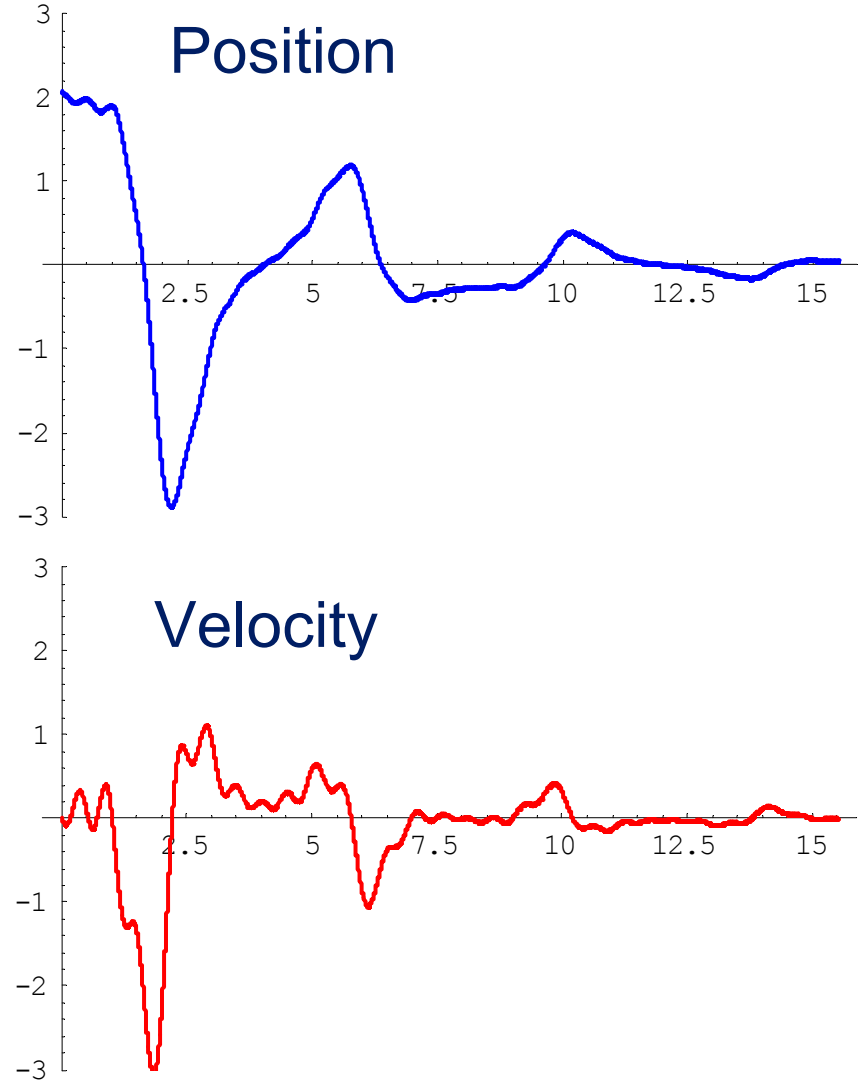
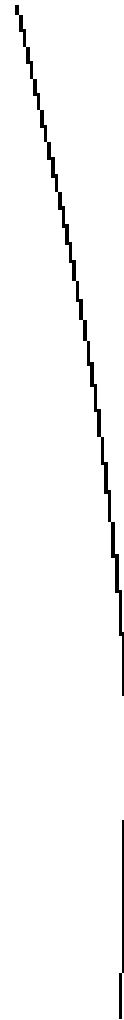
Lumped model: Only a simple oscillation pattern possible

Analogy: Chain



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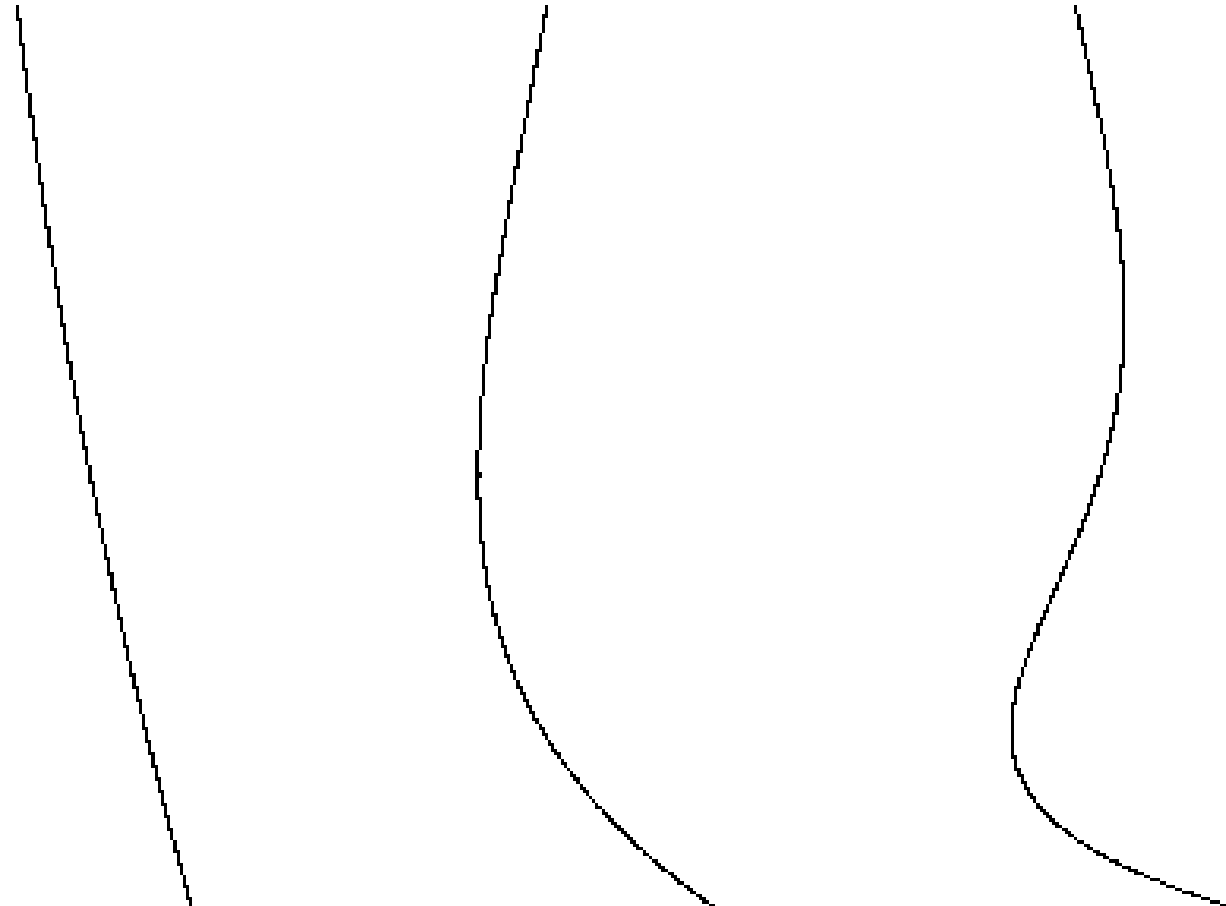
Distributed model: Complex oscillation patterns possible

Analogy: Chain



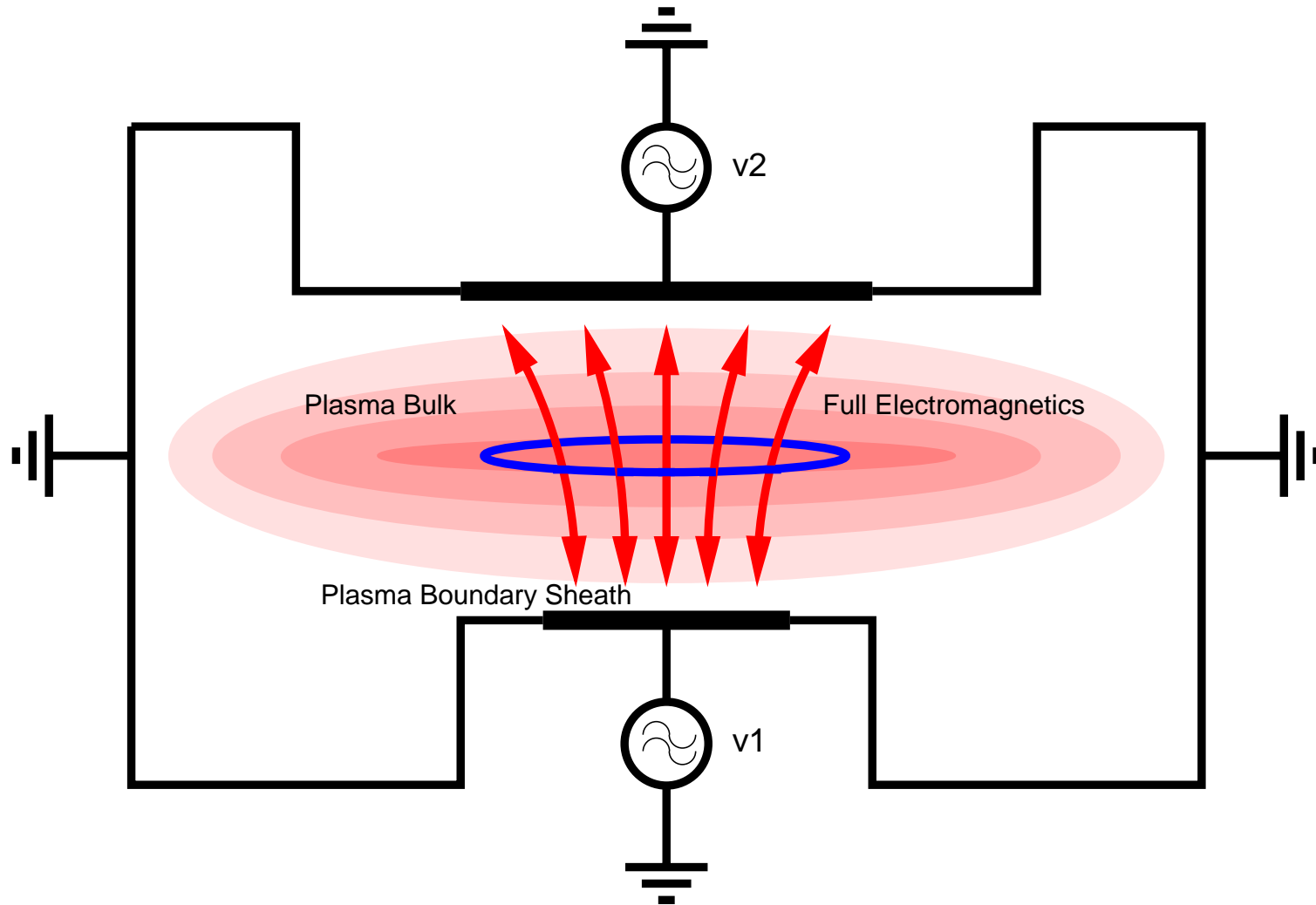
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Eigenmode analysis: Superposition of oscillation modes

General reactor model



Arbitrary geometry, arbitrary number of applied voltages ...



General model: Equations

Definition of a state space:

$$Z = \left\{ |z\rangle = (\mathbf{j}, \mathbf{B}, Q) \mid \nabla \cdot \mathbf{B} = 0, \frac{1}{\mu_0} \nabla \times \mathbf{B} = \mathbf{j}, \mathbf{B} \cdot \mathbf{n} \Big|_{\partial V} = 0 \right\}$$

Dynamical equations:

$$\frac{\partial \mathbf{j}}{\partial t} = \frac{e^2 n_e}{m_e} \mathbf{E} - \nu_c \mathbf{j} \quad \text{Electron equation of motion}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \quad \text{Magnetic induction law}$$

$$\frac{\partial Q}{\partial t} = \mathbf{n} \cdot \mathbf{j} \Big|_{\partial V} \quad \text{Sheath charging}$$

Boundary conditions:

$$\mathbf{n} \times \mathbf{E} \Big|_{\partial V} = -\mathbf{n} \times \nabla (V_S(Q) + \Phi_{RF}) \Big|_{\partial V}$$



General model: Analysis

The state space Z can be interpreted as a Hilbert space, with an inner product motivated by an energy functional

- The concept of orthogonality applies
- The tools of functional analysis can be used

Abstract formulation of the dynamics:

$$\frac{\partial |z\rangle}{\partial t} = T_C |z\rangle + T_D |z\rangle + T_{NL} |z\rangle|z\rangle + u \cos(\omega t) |e\rangle$$

T_C Conservative part: pure oscillation

T_D Dissipative part: damping

T_{NL} Nonlinear harmonics generation

$u \cos(\omega t) |e\rangle$ External excitation



General model: Solution

The equation can be solved by a triple series expansion (power series, Fourier series, eigenmode expansion)

$$|z\rangle = \sum_{n=1}^{\infty} \sum_{k=-n}^n \sum_{l=1}^{\infty} z_{kl}^{(n)} u^n e^{ik\omega t} |r_l\rangle$$

The series coefficients can be found by recursion:

$$z_{kl}^{(1)} = \frac{1}{2(i\omega - p_k)} \langle l_k | e \rangle$$

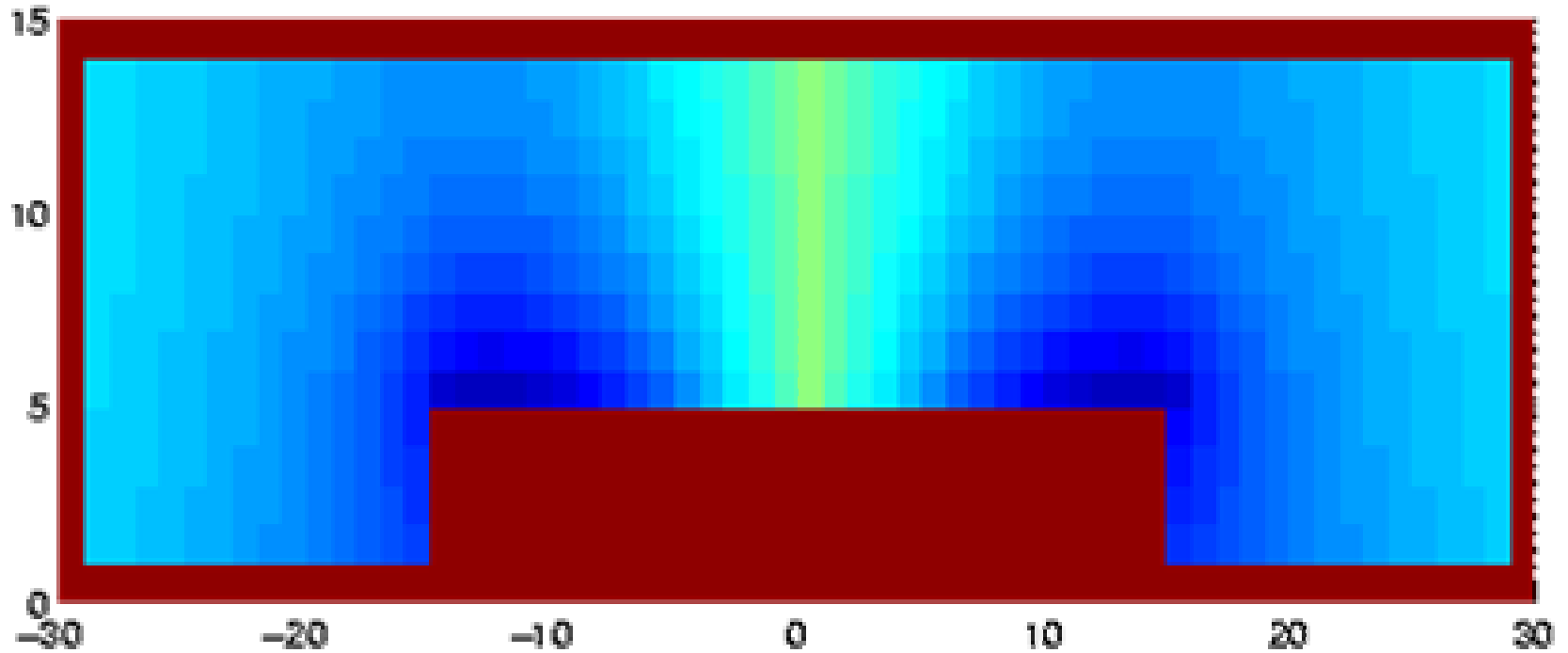
$$z_{kl}^{(n)} = \frac{1}{i\omega - p_k} \sum_{s=1}^{n-1} \sum_{\bar{k}=1}^{\infty} \sum_{\tilde{k}=1}^{\infty} \sum_{|\bar{l}|\leq s, |\tilde{l}|\leq n-s, \bar{l}+\tilde{l}=l} \langle l_k | T_{NL} | r_{\bar{k}} \rangle | r_{\tilde{k}} \rangle z_{\bar{k}\bar{l}}^{(s)} z_{\tilde{k}\tilde{l}}^{(n-s)}$$

Algebra in the solution space of the linear problem:

➔ Numerically tractable even for complicated situations



General model: Example



- Numerical scheme with consistent discretization
- Any 2D-geometry, any electron density distribution
- Tested with analytical solutions for special cases



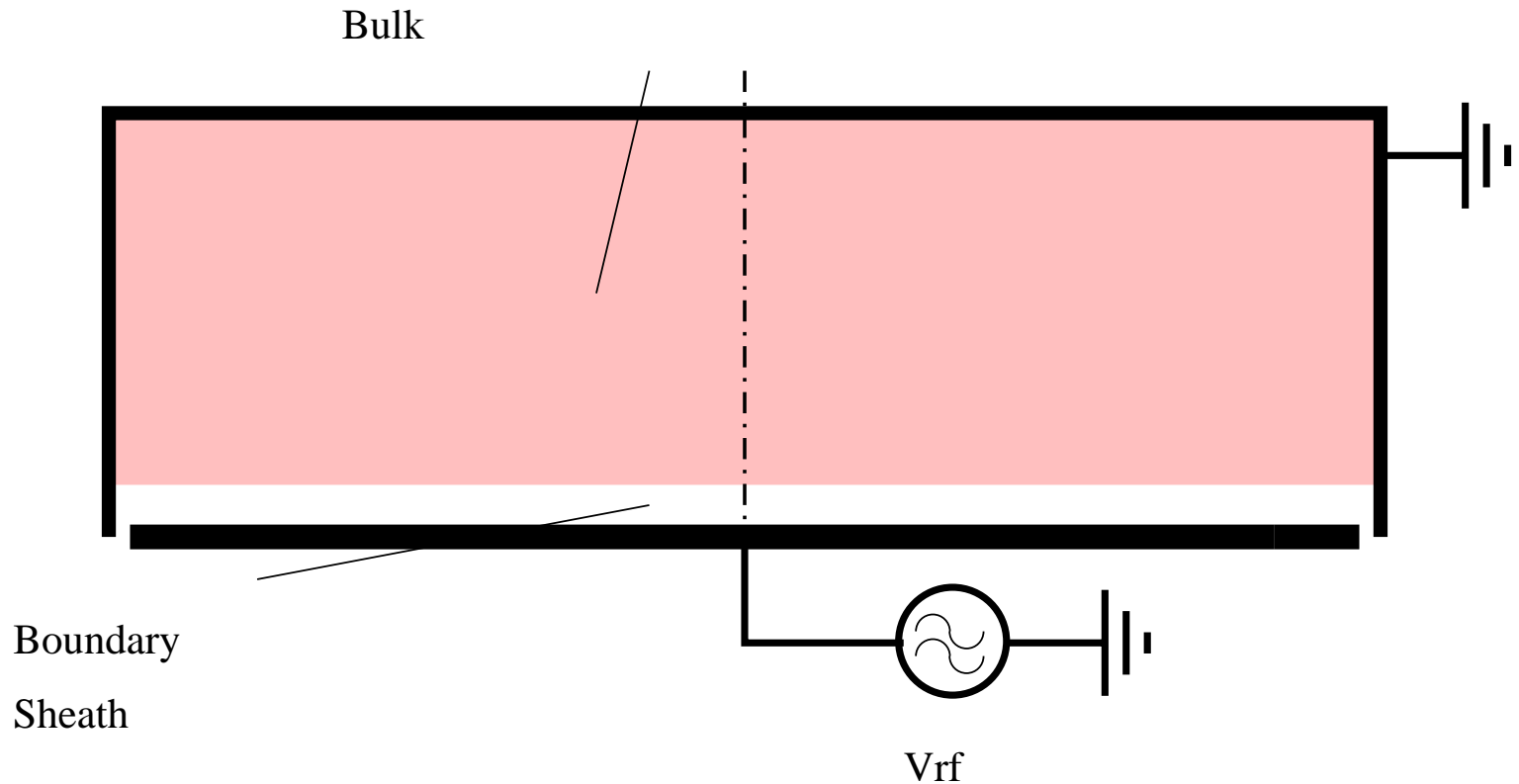


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A Simplified Toy Model

Toy model: Set-Up

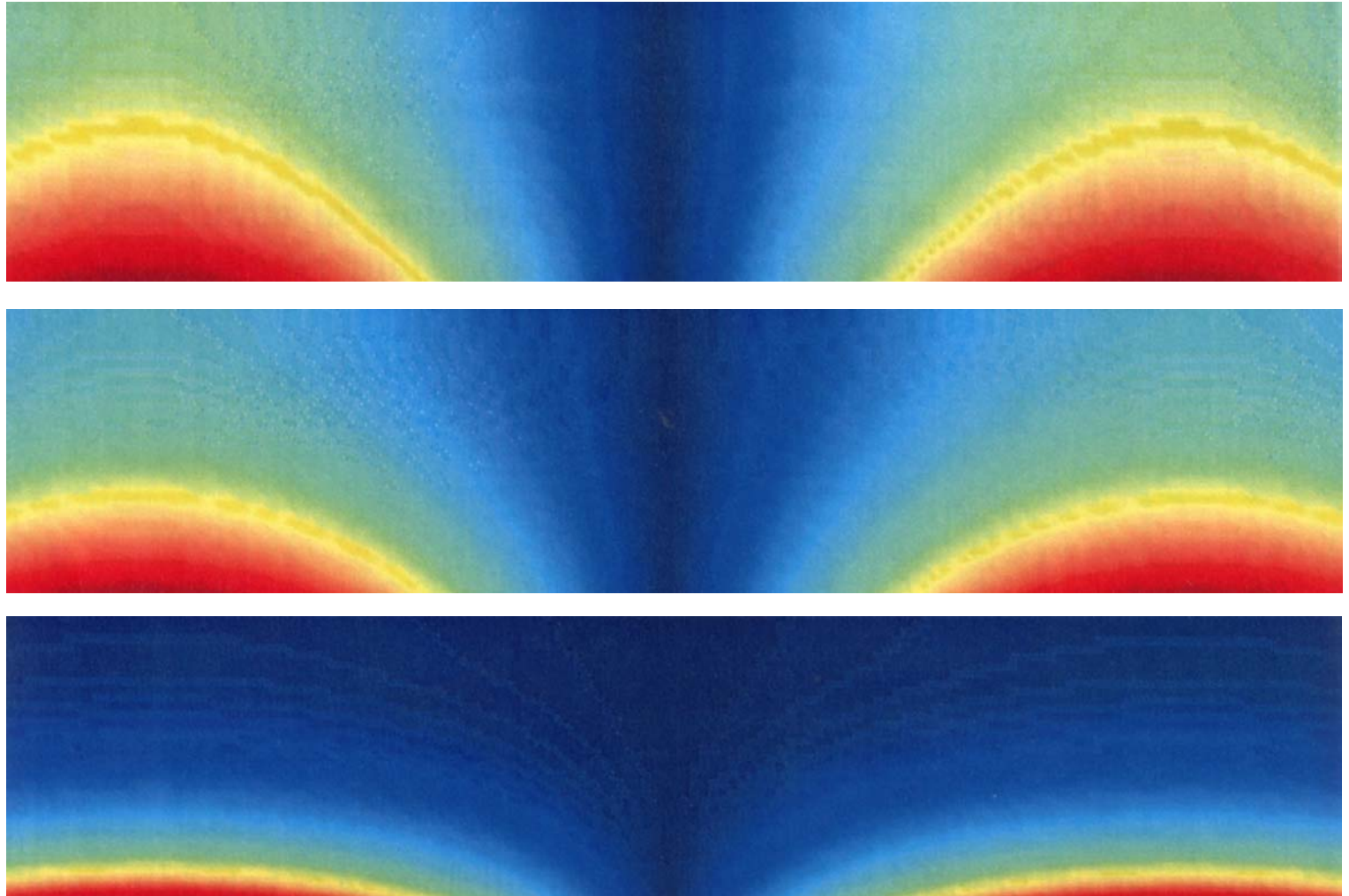


Simplifications to facilitate an analytical treatment:

- Simple axi-symmetric cylinder geometry
- Constant bulk and sheath parameters



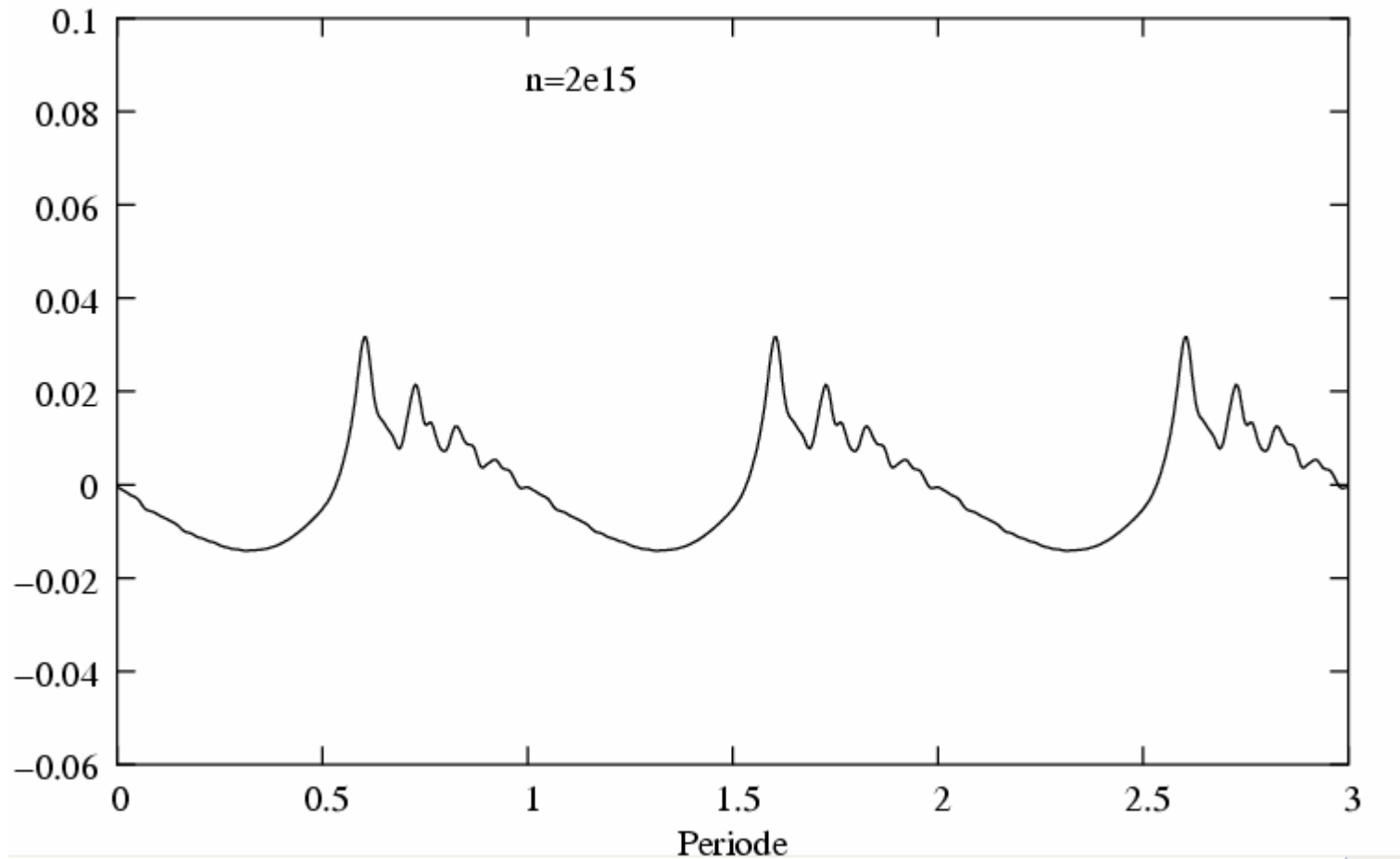
First eigenmode



The eigenmode shape depends on the plasma density



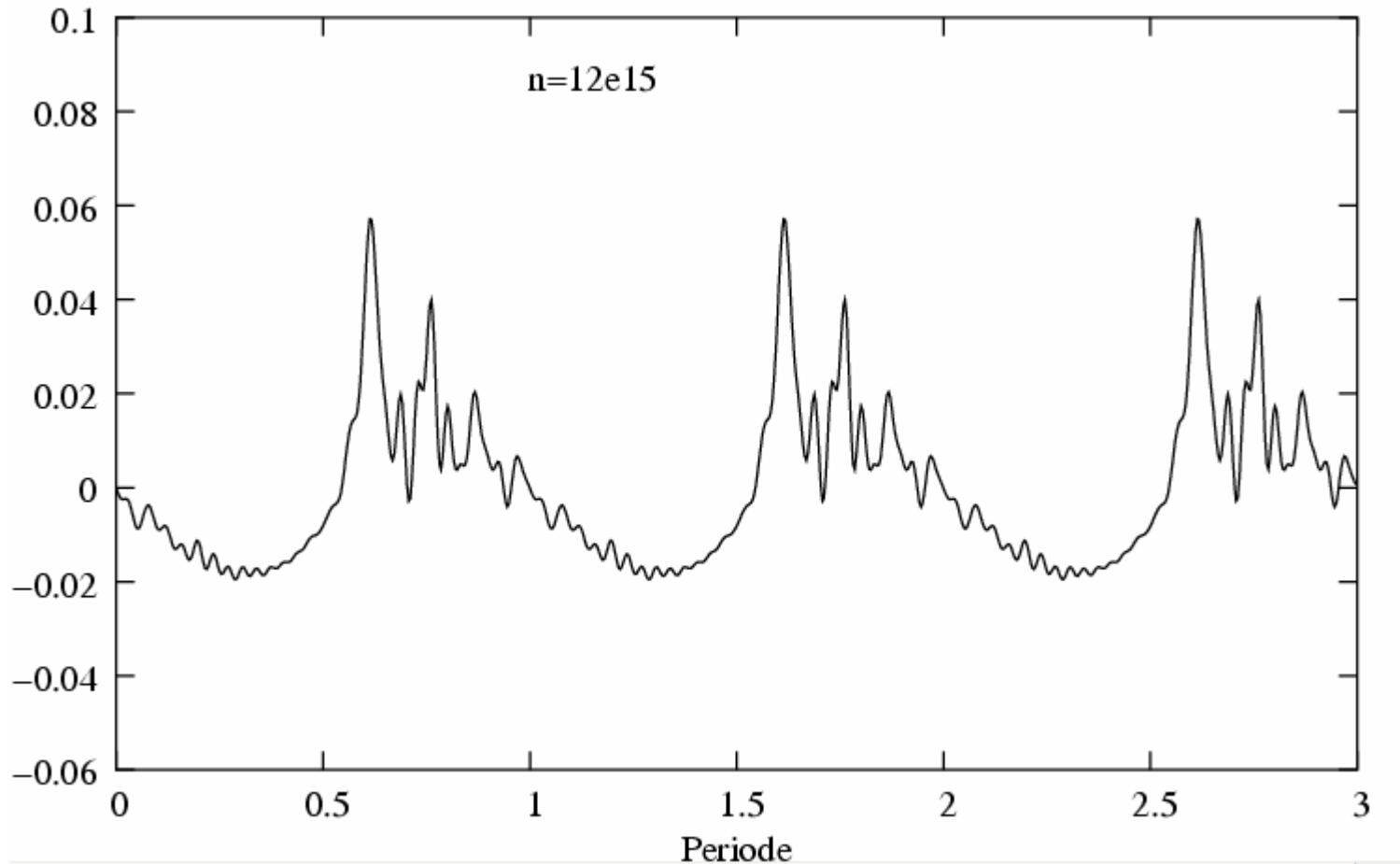
Toy model: Results I



SEERS signal for a small amplitude of the RF excitation



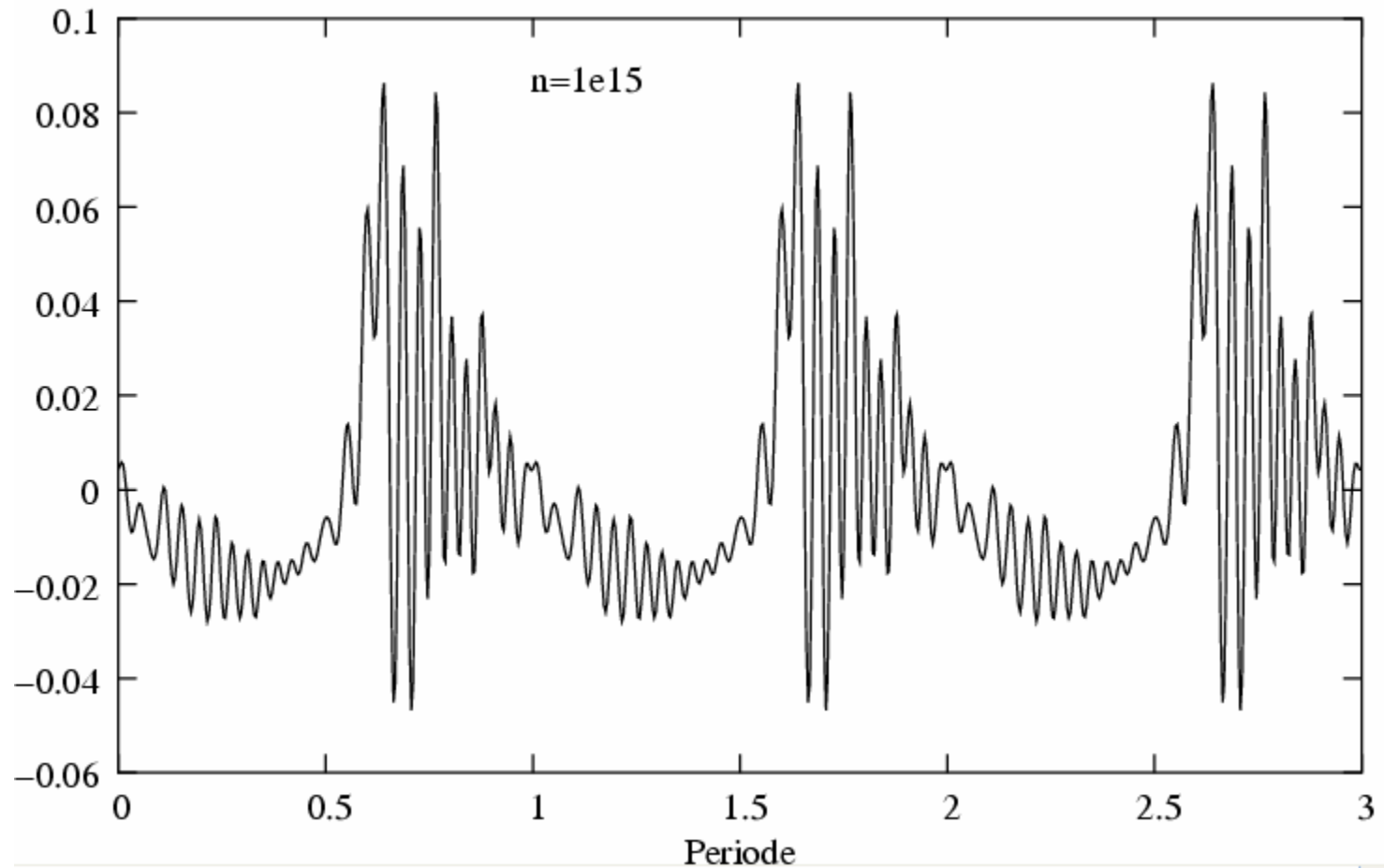
Toy model: Results II



SEERS signal for a medium amplitude of the RF excitation



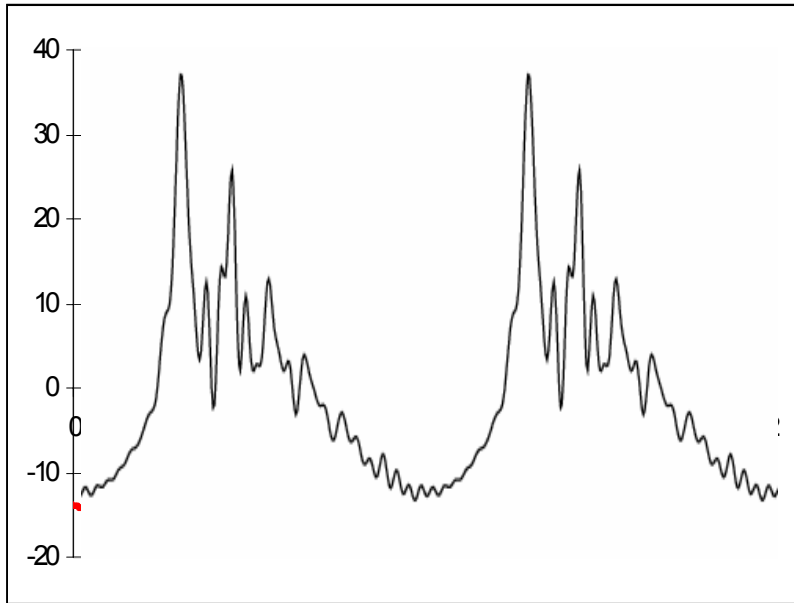
Toy model: Results III



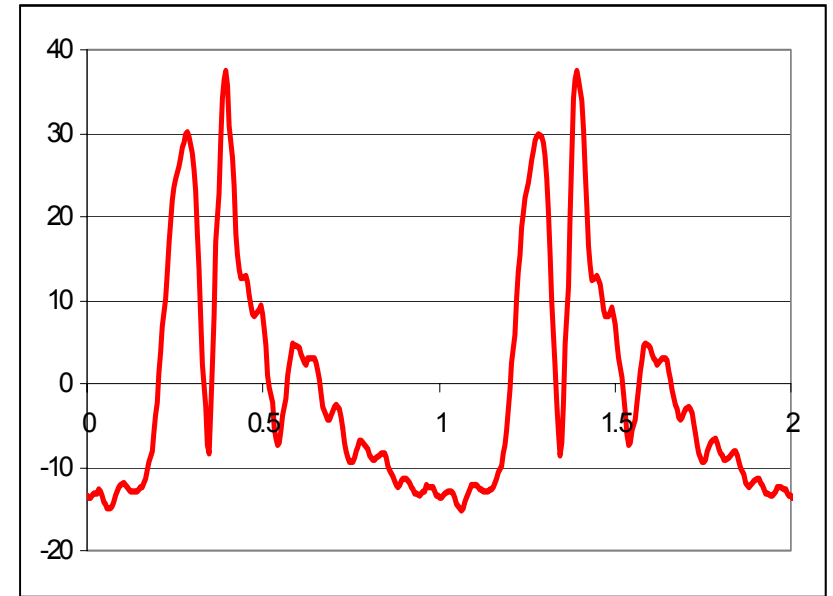
SEERS signal for a large amplitude of the RF excitation



Toy model: Comparing with reality



Simulation



SEERS Sensor-Data

- Good “qualitative” correspondence (form, trends).
- No quantitative agreement yet, but “fine structures”

For a quantitative agreement: Go beyond the toy model!





Perspective

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Perspective

What have we learned?

- Consistent framework for modeling of the SEERS effect, without any restriction on geometry or homogeneity.
- The nonlinear problem can be solved by “doing algebra” with the linear solutions. (In practice, few modes suffice).

Work in progress towards a full implementation:

- Essential elements already in place (nonlinear boundary sheath model, consistent numerical discretization ...).
- Current projects aim to include more physical effects (static magnetic fields, stochastic dissipation ...).
- Implementation into a full reactor simulator planned.
- Comparison with careful experiments is underway.





Thank you!

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