Modeling of SEERS

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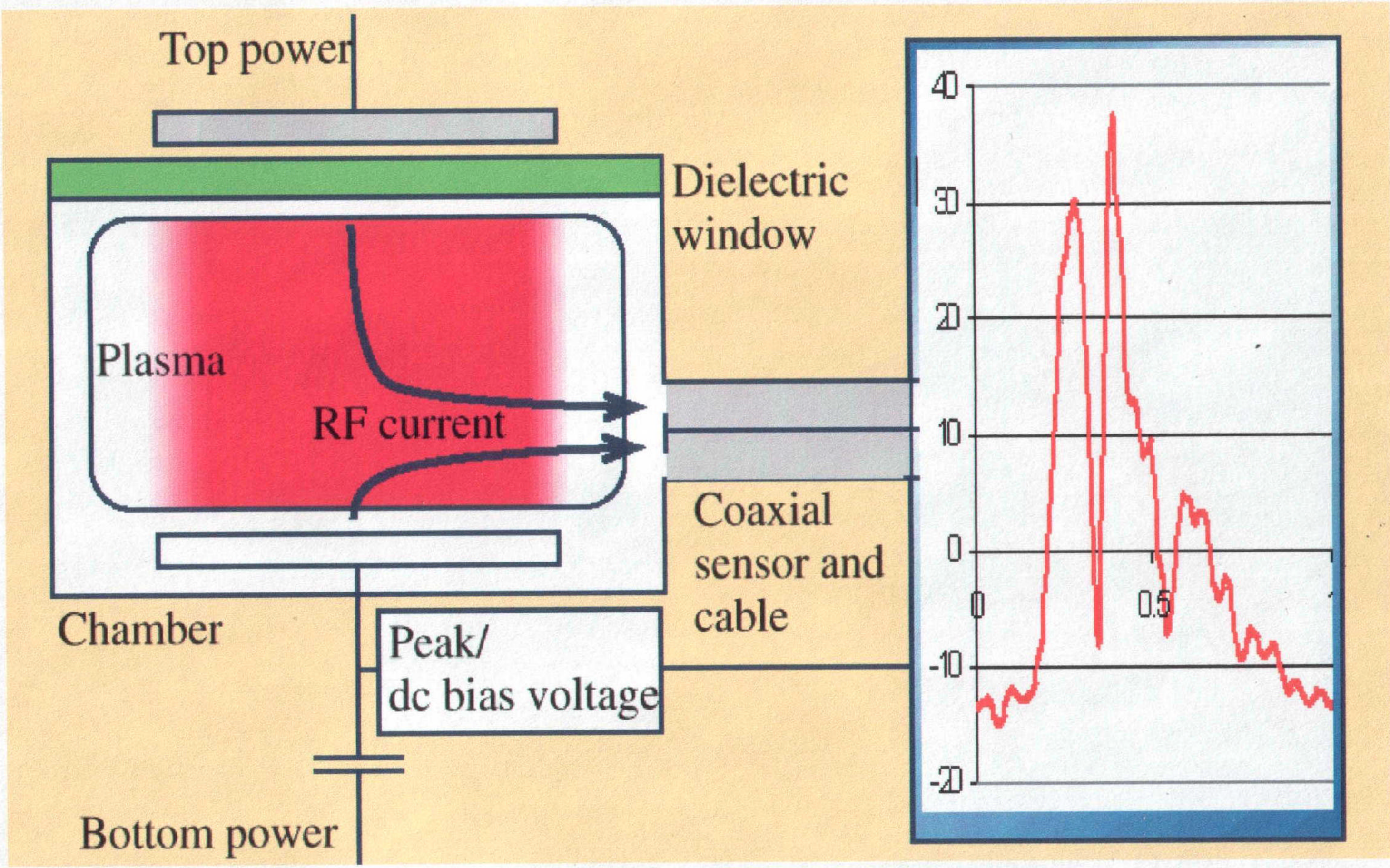
Different views of SEERS

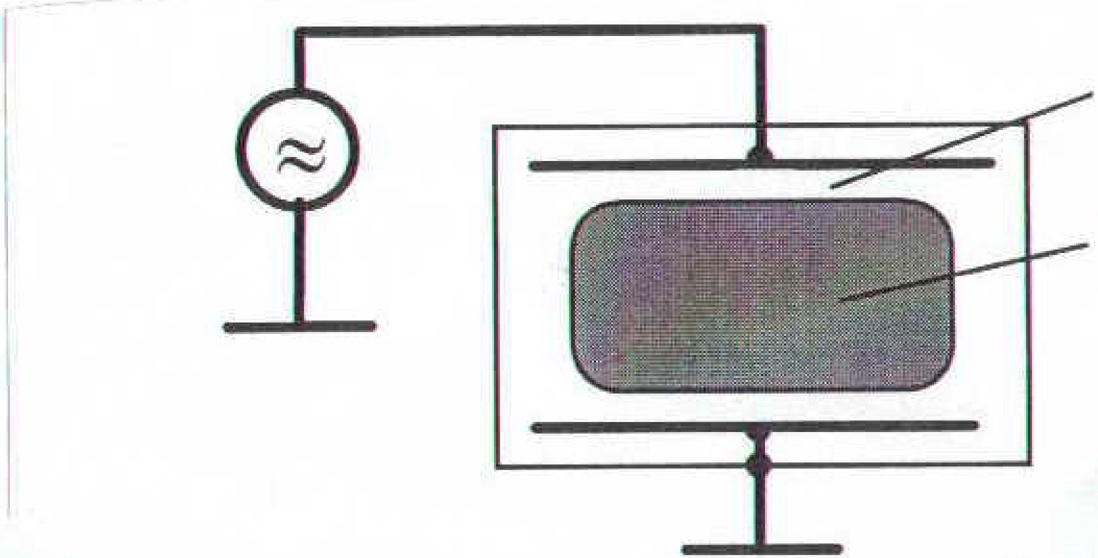
Standard view: SEERS yields three *signals* (n_e, ν_c, P_B)



A deeper understanding of the observed wall current may yield a more exhaustive characterization of the reactor state.

SEERS Effect

SEERS observed the so-called plasma series resonance:



$$\underline{j} = \epsilon_0 \frac{\partial \underline{E}}{\partial t}$$

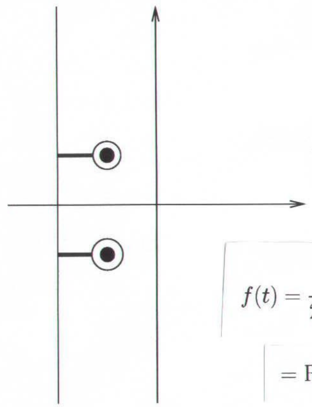
$$\frac{\partial \underline{j}}{\partial t} = \omega_p^2 \epsilon_0 \underline{E} - \nu \underline{j}$$

- Sheath capacitance acts as a capacitor.
- Electron inertia acts as an inductor.
- Electron-neutral collisions (friction) act as a resistor.

A realistic description of SEERS will require a spatially resolved model of the bulk, and a non-linear model of the sheath.

1–D / linear 2 Minutes	1–D / nonlinear 10 Minutes
2–D / linear 20 Minutes	2–D / nonlinear Next Workshop

Laplace Transform

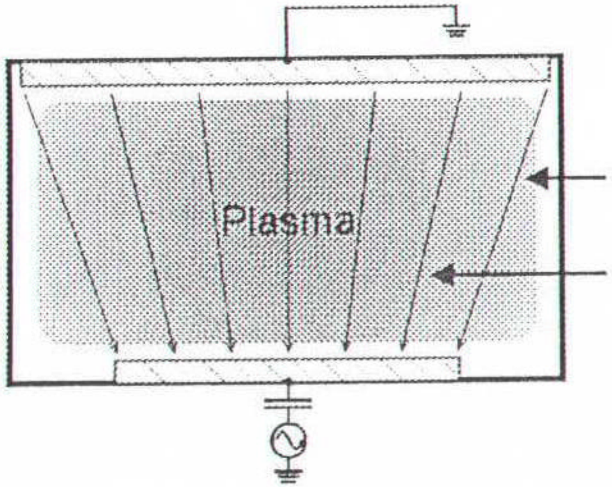


$$F(p) = \int_0^\infty e^{-pt} f(t) dt$$

$$f(t) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} e^{pt} F(p) dp$$

$= \text{Root Contribution} \sim e^{p_k t} + \text{Strongly Damped Residuals}$

Model of the Plasma Bulk



Ground (Area A_G)

Assumed plasma density

Assumed current distribution

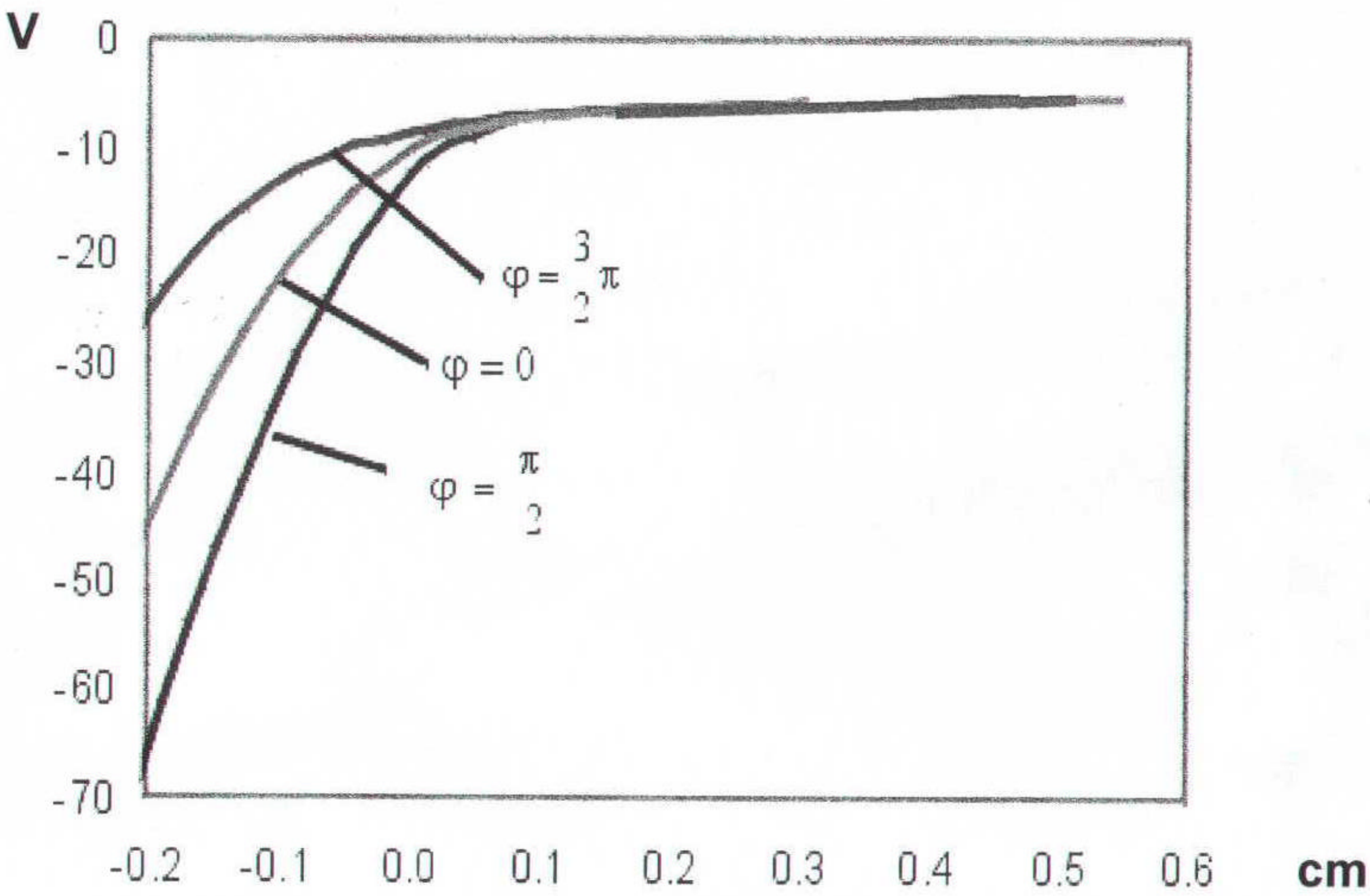
Electrode (Area A_E)

Elements of a "1½ - dimensional" bulk conduction model:

- Assume a "reasonable" plasma density $n(\underline{r}) = n_{AV} * g(\underline{r})$
- Assume a "reasonable" current distribution $j_x(x) = I / A(x)$
- Relate field and current density via $d\underline{j}/dt = \omega_p^2 \epsilon_0 \underline{E} - \nu_c \underline{j}$
- The resulting I-V-relation is that of a "lossy inductance":

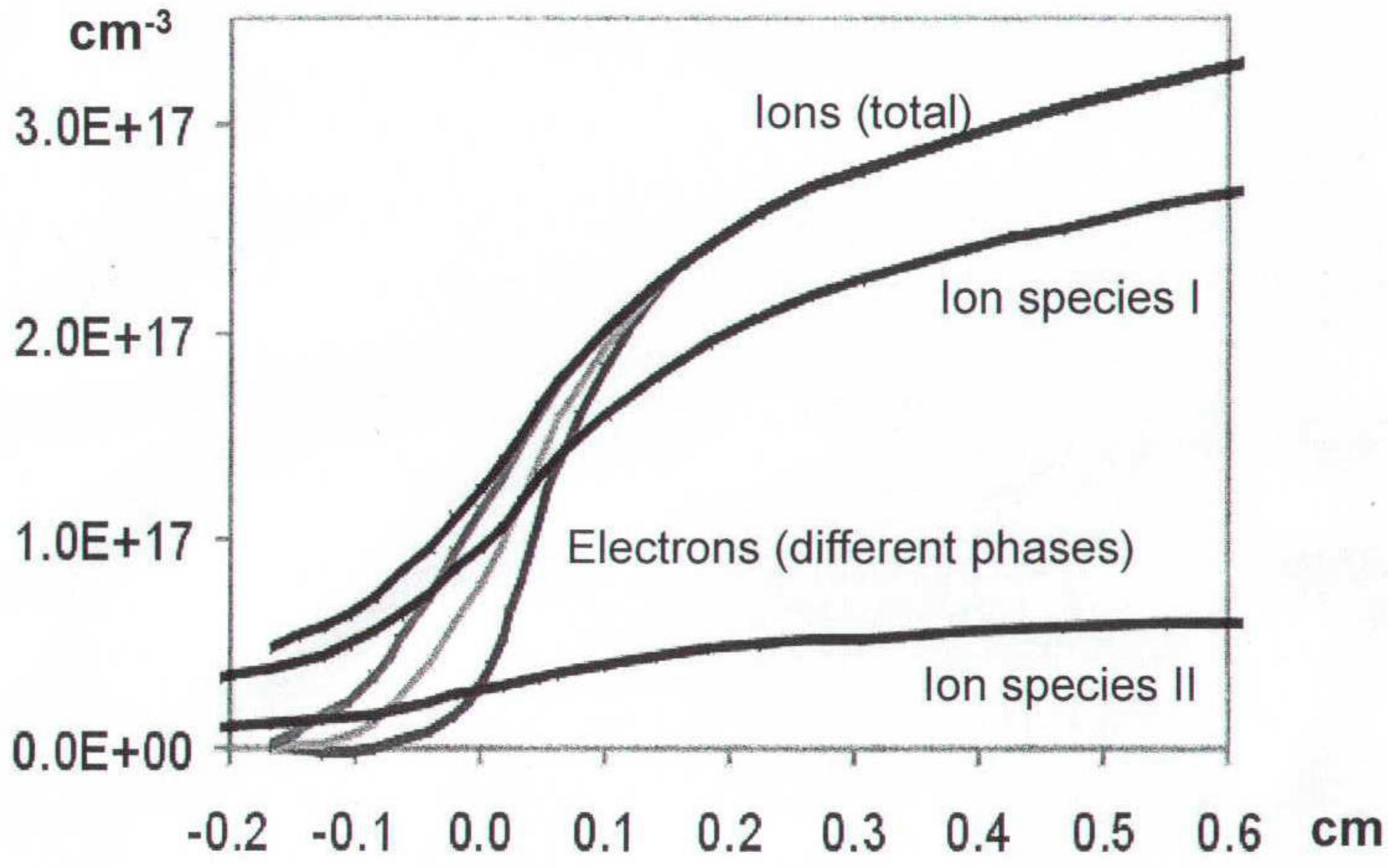
$$L * dI/dt + R * I = V$$

Potential Distribution in an RF Sheath



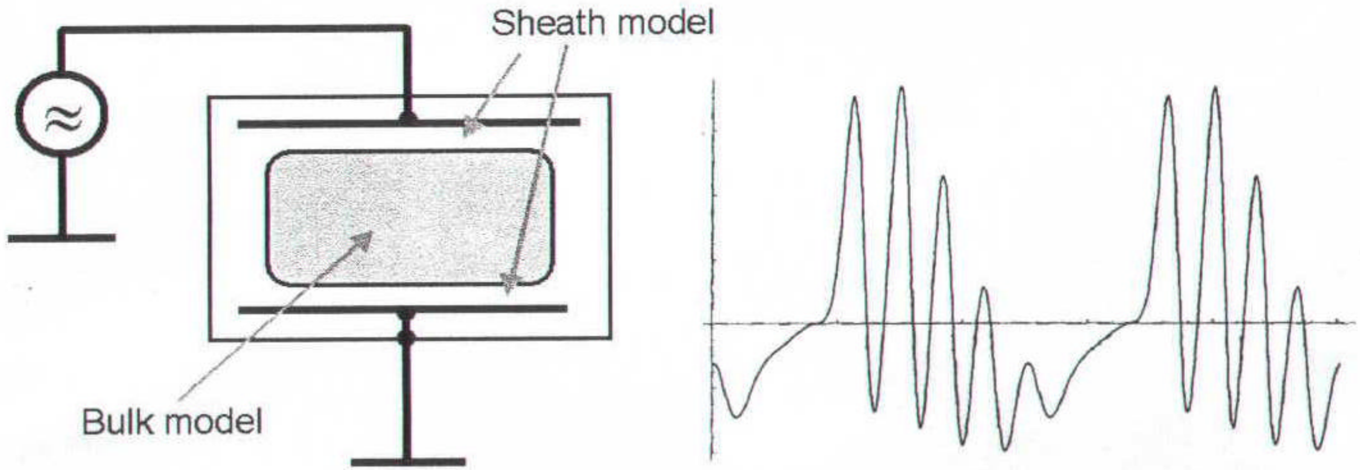
Electrical potential in a “generic” two-species sheath.

Particle Densities in an RF Sheath



Electron and ion densities in a “generic” two-species sheath.

Putting Sheath and Bulk Model Together

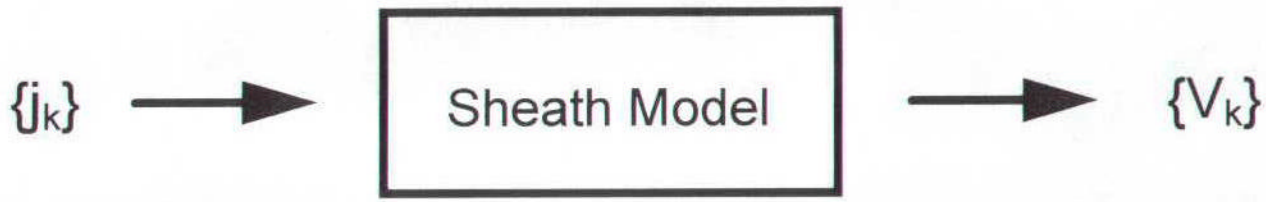


Features of the "1½ - dimensional" SEERS model:

- Comprised of the (very simple) linear bulk model and two copies of the (sophisticated) nonlinear sheath model
- The asymptotic limit cycle assumed under RF excitation is calculated using the method of harmonic balance.
- Feasible, numerically stable and efficient (<10s on a PC)
- The self-excited electron resonance is clearly visible.

Now, that all tasks are completed, what does the model do?

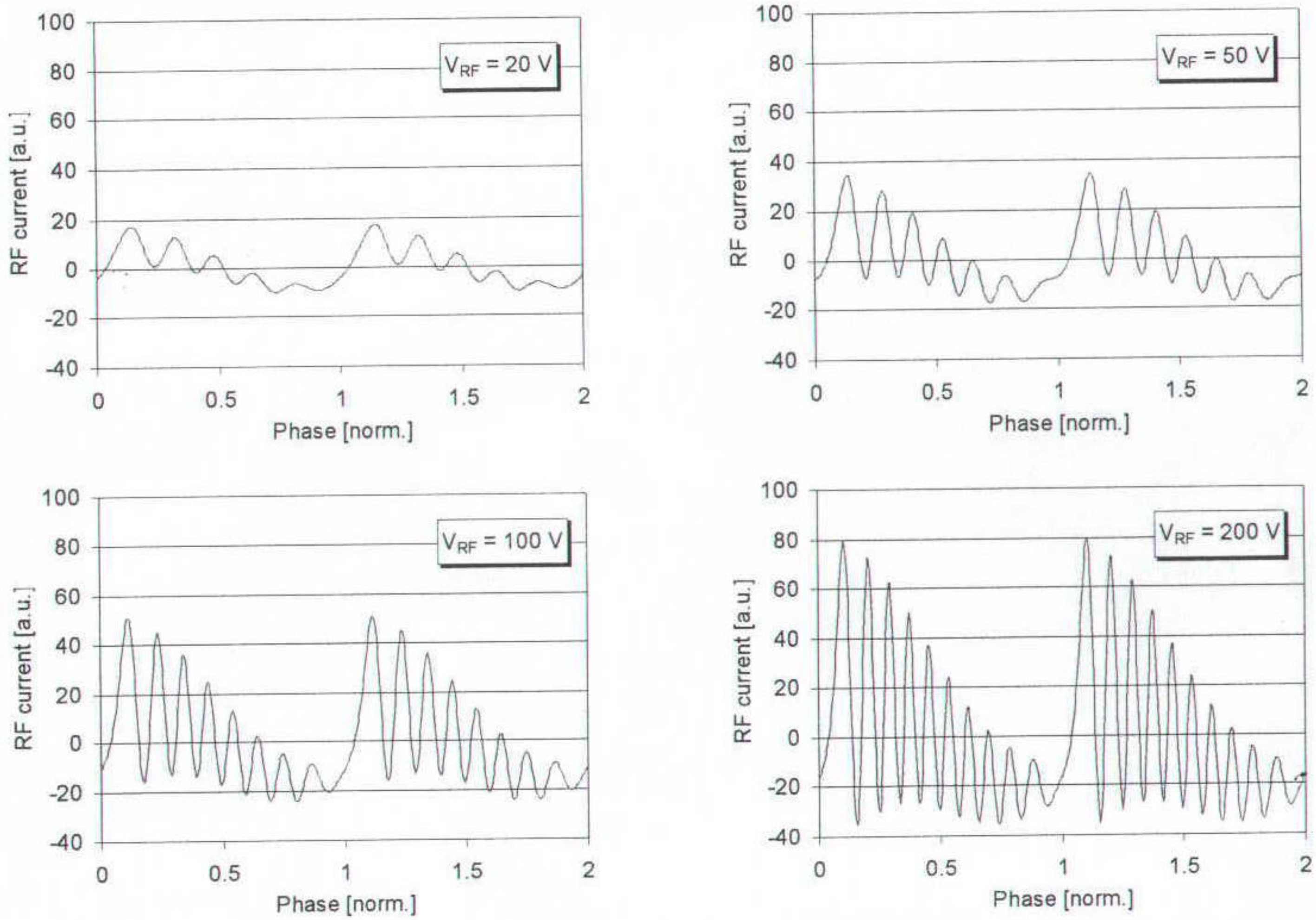
- It takes the Fourier coefficients j_k of the RF current
- It calculates the field and density distribution in the sheath
- It gives back the Fourier coefficients V_k of the sheath voltage



The model is not just a „nonlinear sheath capacitance", because the charge distribution itself is also influenced by the current Fourier coefficients:

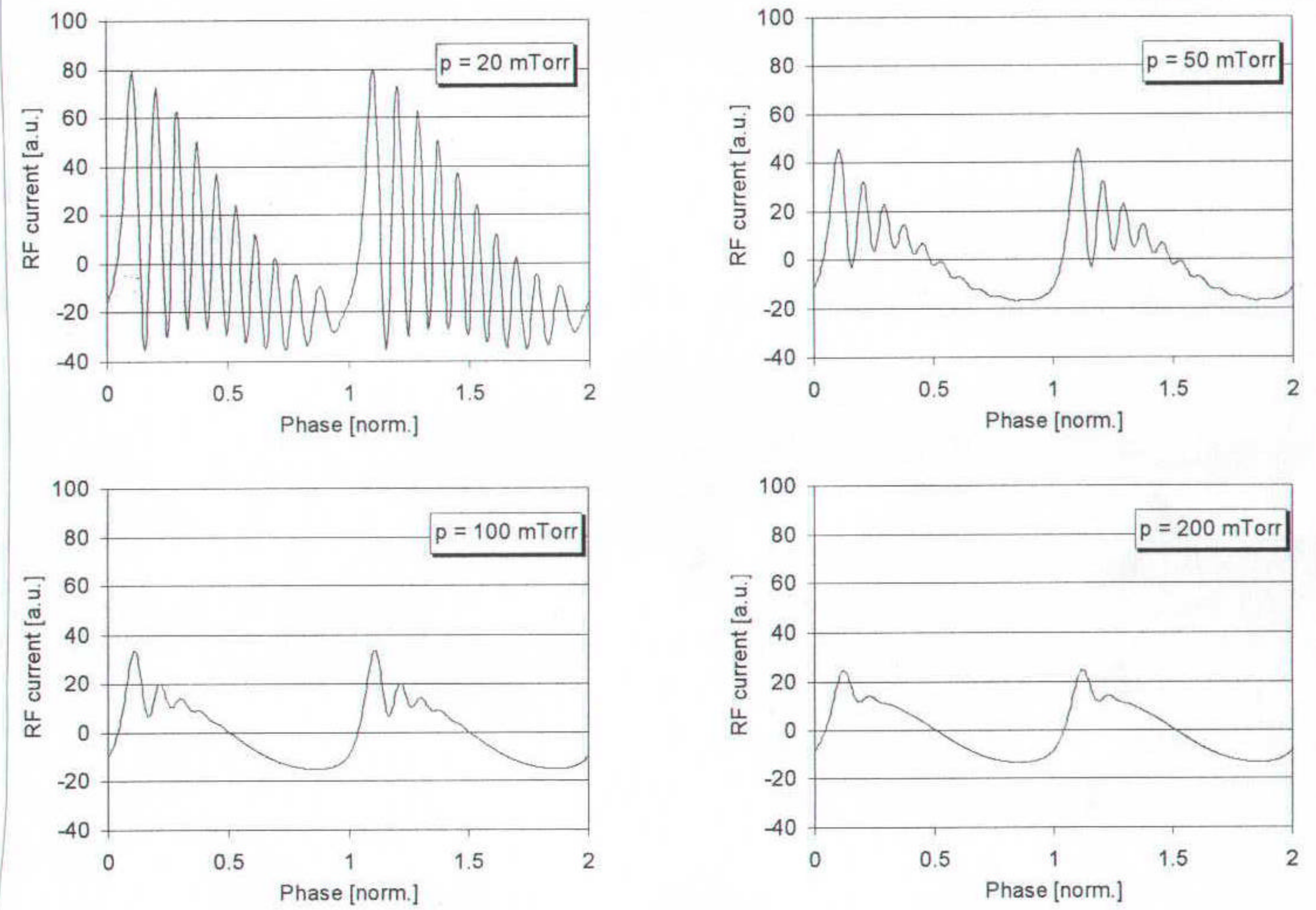
$$V(t) = V(Q(t), \{j_k\}) \neq V(Q(t))$$

Parameter Study: Increasing RF Excitation



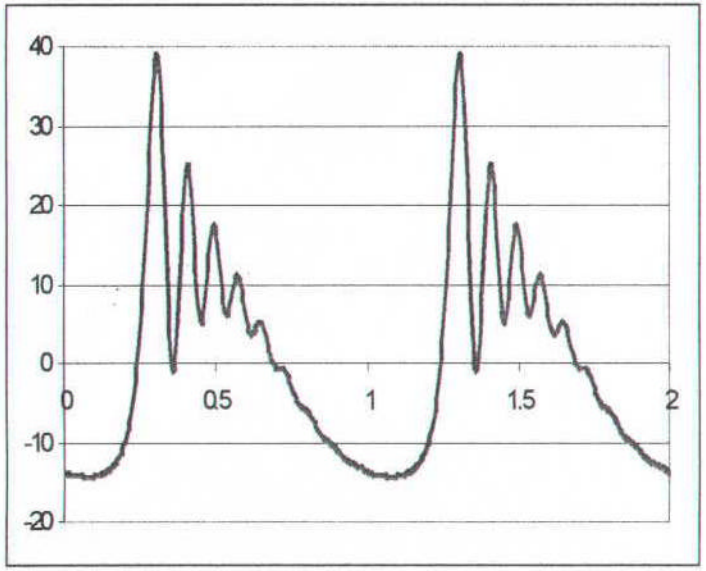
Current amplitude and oscillation frequency grow with excitation.

Parameter Study: Increasing Gas Pressure

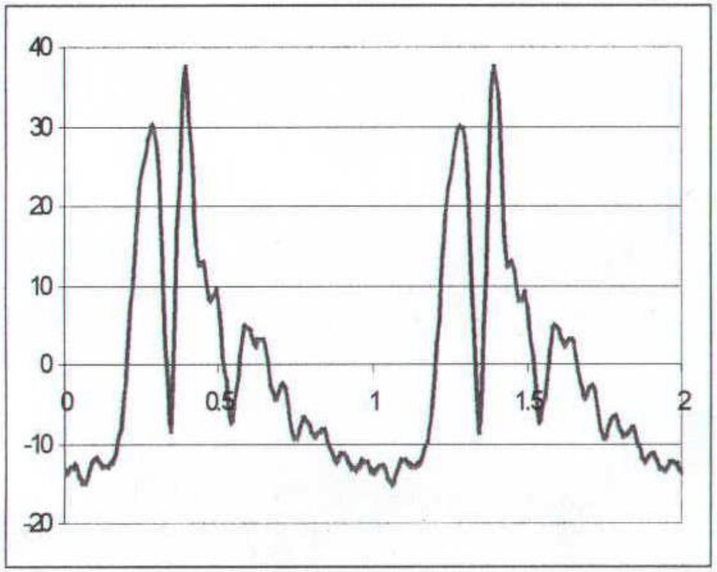


SEERS oscillations are increasingly damped with pressure.

Comparison with Actual SEERS Data



SEERS Simulation



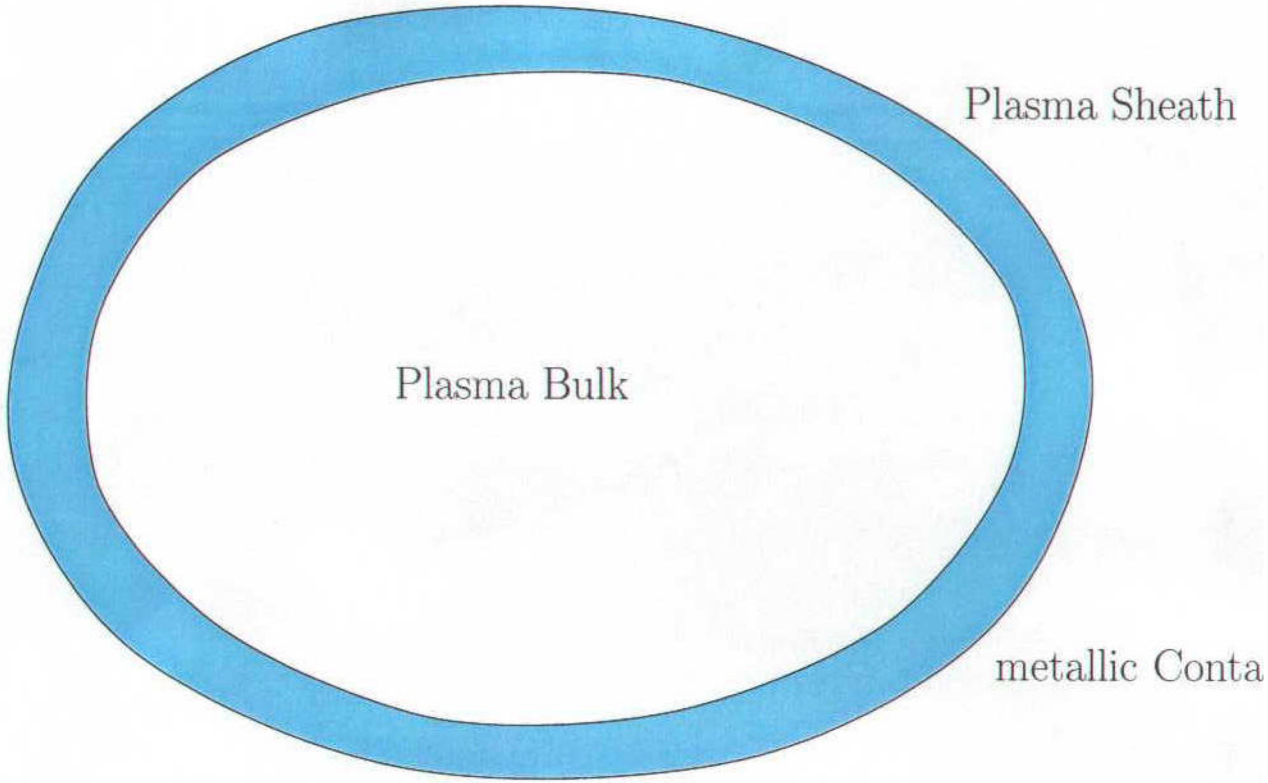
SEERS Data

Comparison shows a "good qualitative agreement":

- Good reproduction of the saw-tooth structure
- Poor reproduction of the detailed structures
- Good reproduction of the physical trends
- Poor quantitative agreement (roughly off by 50%)

1–D / linear ✓	1–D / nonlinear ✓
2–D / linear now	2–D / nonlinear Next Workshop

Situation



Wanted:
Eigenmodes of the sytem for an arbitary initial condition.

Bulk Equations

Maxwell Equations:

$$\frac{1}{\mu_0}\nabla \times \vec{B} = \vec{j}$$

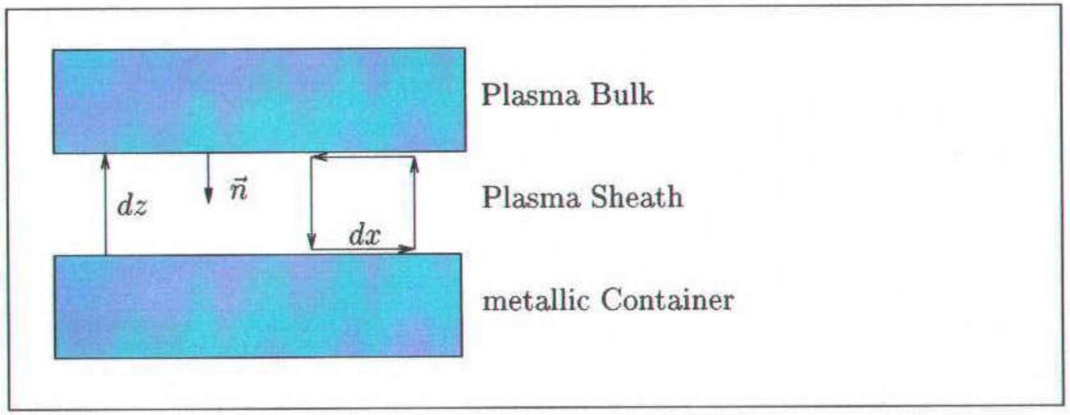
$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Equation of Motion for Electrons:

$$\frac{\partial \vec{j}}{\partial t} = \frac{e^2 n_e}{m_e}\vec{E} - \nu_C \vec{j}$$

Sheath Equations



$$C_S \frac{\partial V_S}{\partial t} = \vec{j} \cdot \vec{n}$$

$$\oint \vec{E} \cdot d\vec{s} = 0$$

$$\frac{\partial V_S}{dx} = E_{\tan}$$

Normalization

Normalization Basis

L : Typical Dimension

n : Typical Density

C : Typical Sheath Capacity

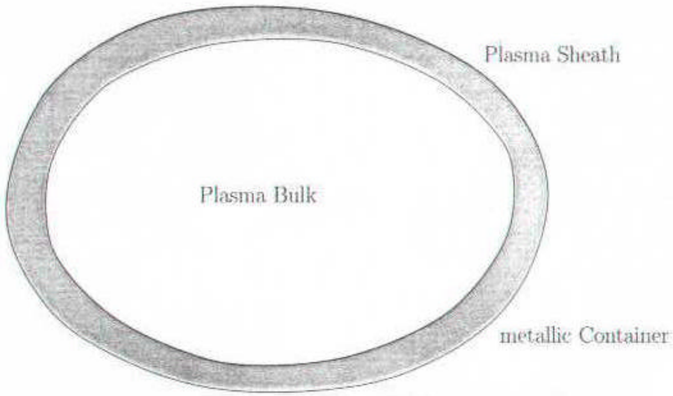
$\longrightarrow \Omega$: Linear SEERS Frequency

Characteristic Numbers

$$R_m = \frac{e^2 n_e \mu_0 H^2}{4\pi^2 m_e}$$ Magnetic Reynolds Number

$$\nu = \frac{\nu_C}{\Omega}$$ Normalized Collision Frequency

Normalized Equations



$$\nabla \times \vec{B} = \vec{j}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -R_m \frac{\partial \vec{B}}{\partial t}$$

$$\frac{\partial \vec{j}}{\partial t} = n_e \vec{E} - \nu_C \vec{j}$$

$$\vec{n} \cdot \vec{j} = C_S \frac{\partial V_S}{\partial t} = \nabla \cdot \vec{j}_F = -\nabla \cdot \Big(\vec{n} \times$$

$$\vec{n} \cdot \vec{B} = 0$$

$$\vec{n} \times \vec{B} = -\vec{j}_F$$

$$\vec{n} \times \vec{E} = -\vec{n} \times \nabla V_S$$

Energy Balance

$$\frac{\partial}{\partial t}\left(\int_V dv \frac{1}{2n_e} j^2 + \int_V dv \frac{1}{2} R_m B^2 + \int_{\partial V} df \frac{1}{2} C_S V_S^2\right) = -\int_V dv \frac{\nu_C}{n_e} j^2 \leq 0$$

Conclusions:

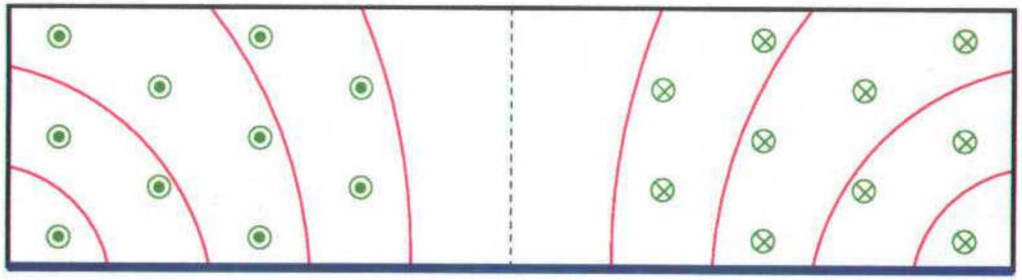
- The system is dissipative
- Consistency of Equations / Boundary Conditions
- $\vec{B}$, $\vec{j}$ und V_S are adequate Variables

Solution Strategy:

- Identify the Problem: Initial Value / Boundary Value Problem
- Apply Laplace Transform
- Look for Eigenvalues

A Toy Model

- Cylindrical Geometry
- Electron Density constant
- Electrode Capacity constant
- Wall Capacity infinite



$$R_m \frac{\partial B_\varphi}{\partial t} + \left(\frac{\partial}{\partial t} + \nu_C \right) \mathcal{H} B_\varphi = 0$$

$$C_S \frac{\partial V_S}{\partial t} + \frac{1}{r} \frac{\partial}{\partial z} (r B_\varphi) = 0$$

$$\left(\frac{\partial}{\partial t} + \nu_C \right) \frac{\partial}{\partial z} B_\varphi + \frac{\partial}{\partial r} V_S = 0$$

$$\mathcal{H} := -\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} r \right) - \frac{\partial^2}{\partial z^2}$$

Equations to be solved

Bulk Equation:

$$pR_mB_\varphi + (p + \nu_C)\,\mathcal{H}B_\varphi = 0$$

Sheath Equation:

$$pC_SV_S + \frac{1}{r}\frac{\partial}{\partial z}\left(rB_\varphi\right) = 0$$

Motion Equation:

$$(p + \nu_C)\,\frac{\partial}{\partial z}B_\varphi + \frac{\partial}{\partial r}V_S = 0$$

Operator:

$$\mathcal{H} := -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}r\right) - \frac{\partial^2}{\partial z^2}$$

Solving the Problem

$$B_\varphi = \sum_{l=0}^{\infty} b(z) J_1\left(\frac{\lambda_{0l}}{R} r\right)$$

$$V_S = \sum_{l=0}^{\infty} v_l J_0\left(\frac{\lambda_{0l}}{R} r\right)$$

λ_{0l} : zeroes of $\frac{\partial}{\partial r}(r$

- J_1 : Eigenfunctions of the radial part of $\mathcal{H}$
- most Boundary Conditions fullfilled naturally

Result:

- Linear Differential Equation in z with constant complex coefficients
- Nonlinear Characteristic Equation for p

Characteristic Equation

$$0 = 1 + \frac{p(\nu_C + p)R\sqrt{\frac{\lambda^2}{R^2} + \frac{pR_m}{\nu_C + p}} \tanh \sqrt{\frac{\lambda^2}{R^2} + \frac{pR_m}{\nu_C + p}}}{\lambda}$$

ν_C : Collision Frequency

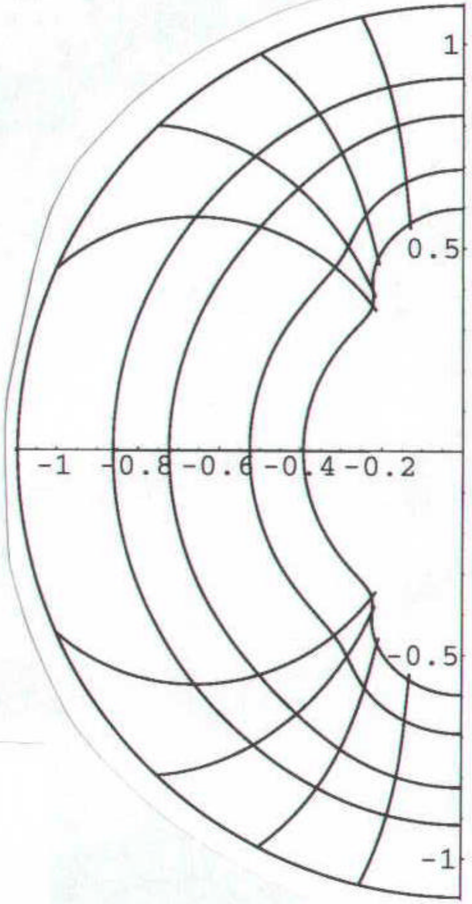
R_m : Magnetic Reynolds Number

R : Aspect Ratio

λ : Zeroes of $\frac{\partial}{\partial r}\left(r J_1(r)\right)$

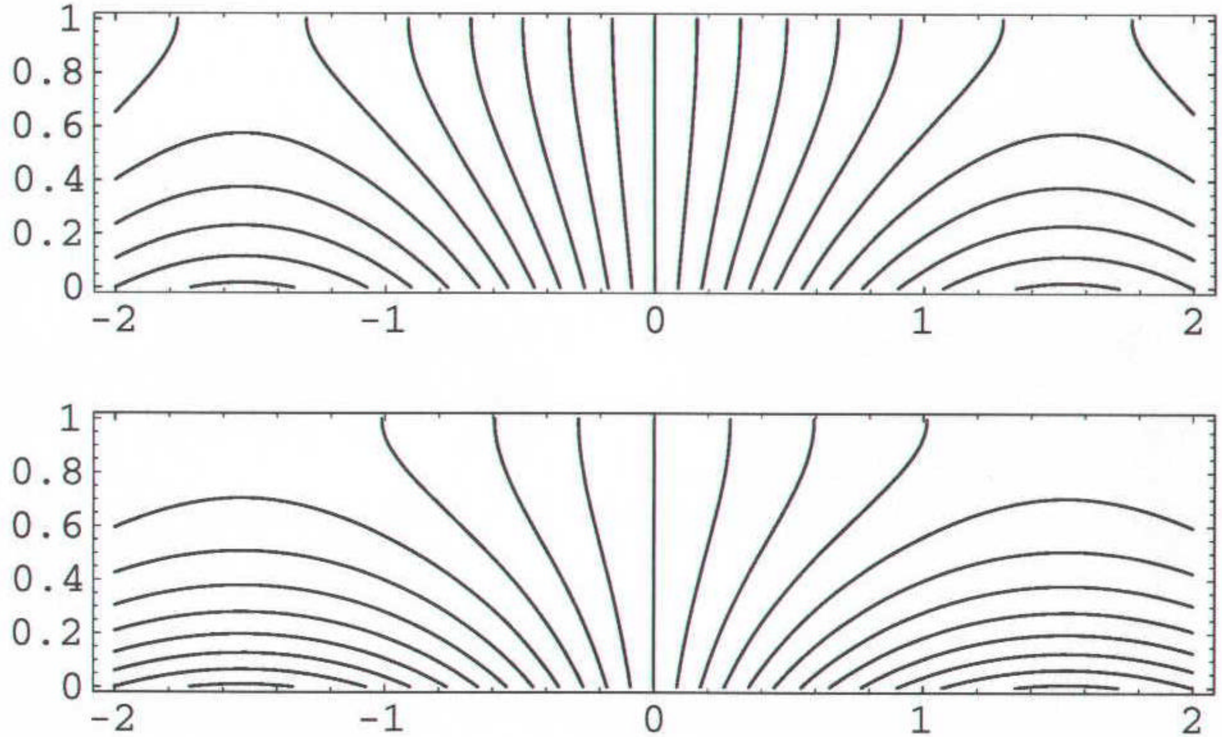
Location of the Roots

Location of the roots of the first resonance with (H/R = 1/2), as depending on the normalized electron collision rate ν and the magnetic Reynolds number R_m.

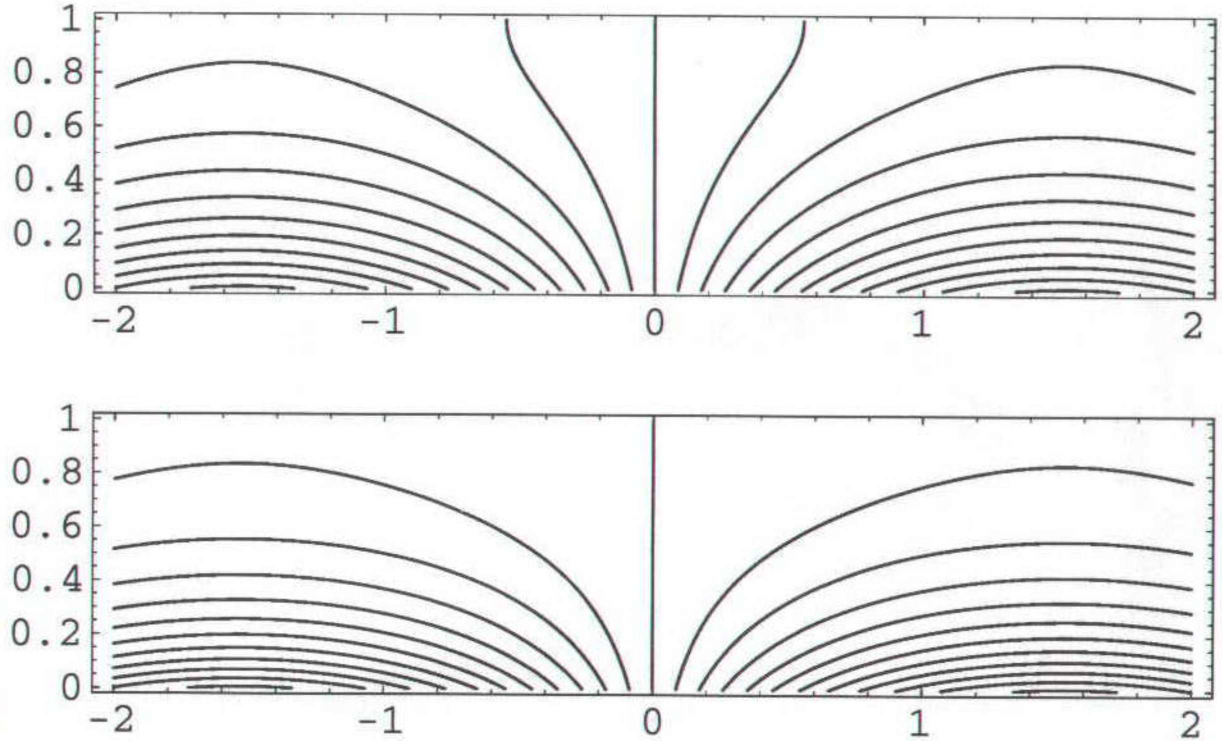


$$0 = 1 + \frac{p(\nu_C + p) R \sqrt{\frac{\lambda^2}{R^2} + \frac{p R_m}{\nu_C + p}} \tanh \sqrt{\frac{\lambda^2}{R^2} + \frac{p R_m}{\nu_C + p}}}{\lambda}$$

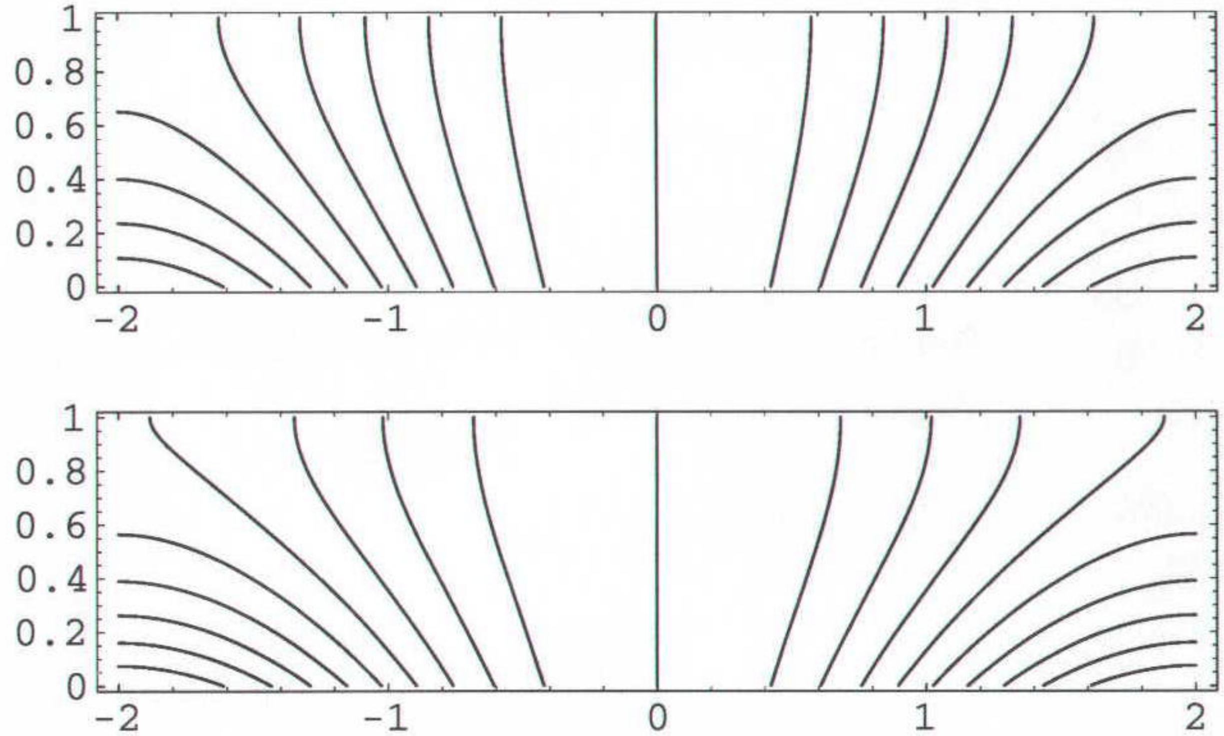
Magnetic Field @ $R_m = 0$ and $R_m = 2$



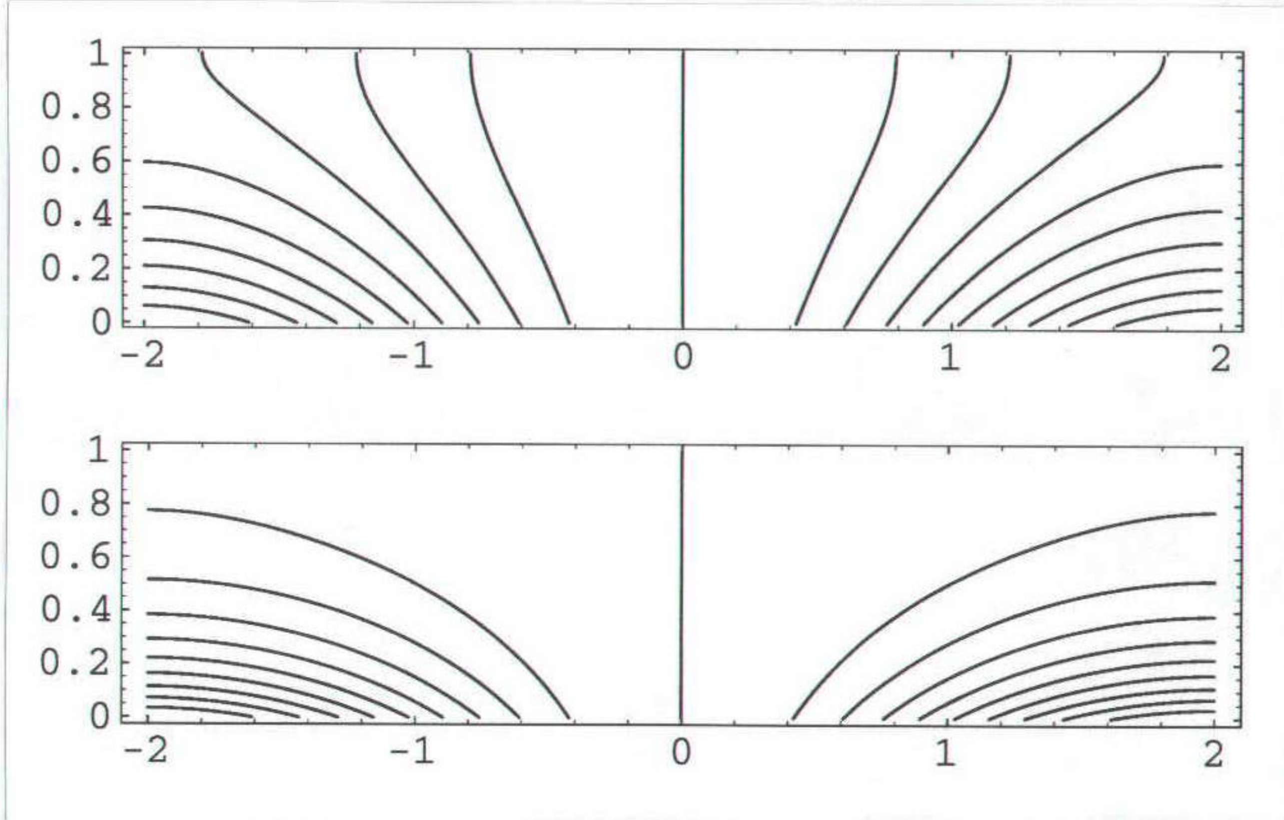
Magnetic Field @ $R_m = 5$ and $R_m = 10$



Current Density @ $R_m = 0$ and $R_m = 2$



Current Density @ $R_m = 5$ and $R_m = 10$



Summary and Conclusion